Lecture 8
2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

General theory

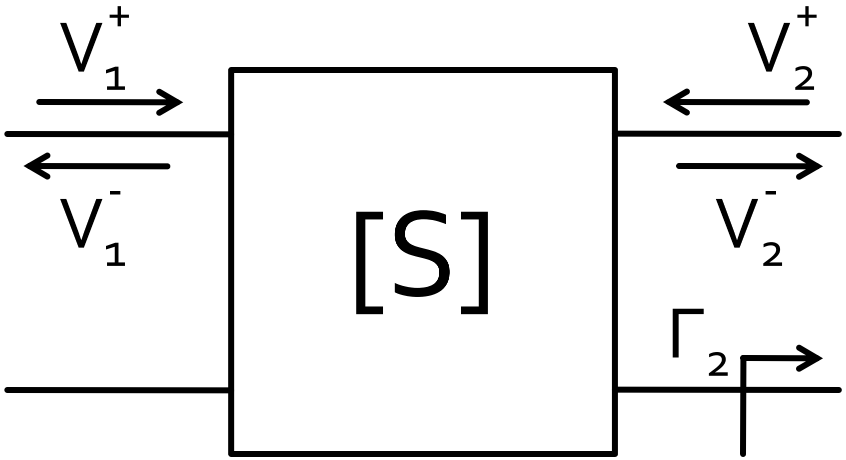
Microwave Network Analysis

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Scattering matrix – S

- Scattering parameters



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=\left[S_p\right]\cdot[a]$$

$$\left[S_p\right]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

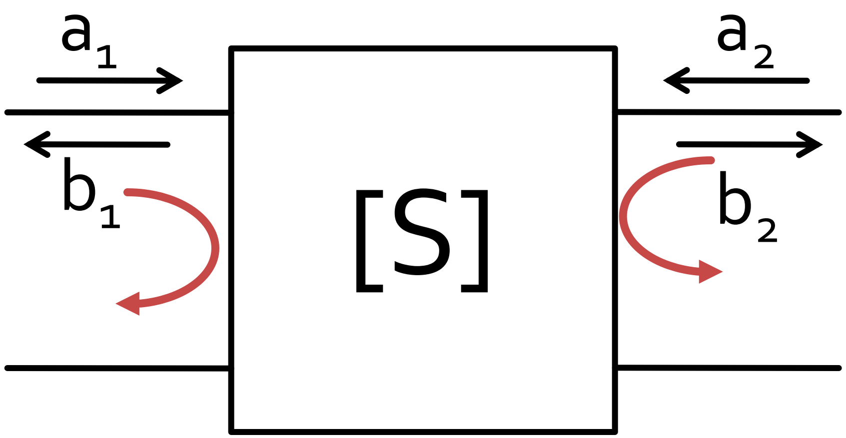
$$Z_{0i}=Z_{Ri}=R_0,\forall i$$

$$R_0=50\Omega$$

$$\left[S_p\right]\equiv[S]$$

- **they coincide!!!**

Scattering matrix – S

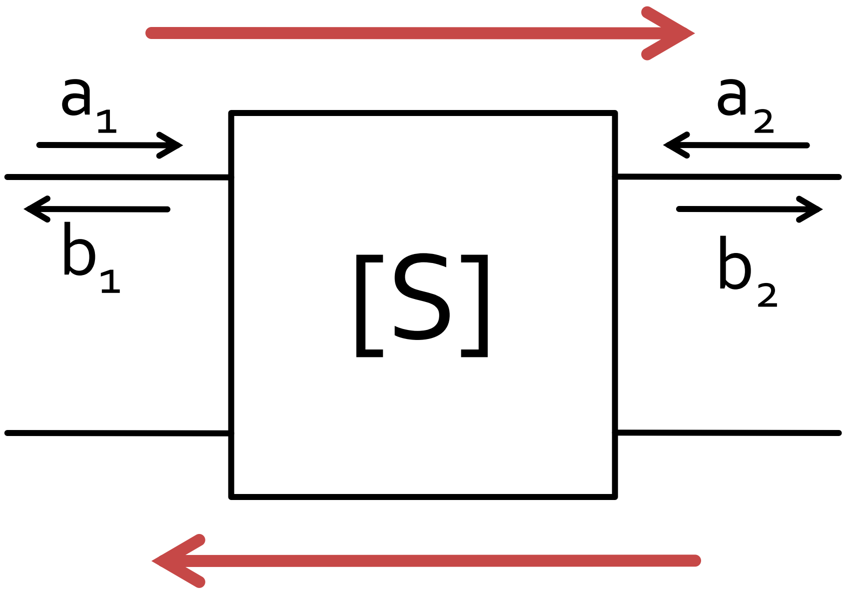


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \qquad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- S_{11} and S_{22} are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

Scattering matrix – S

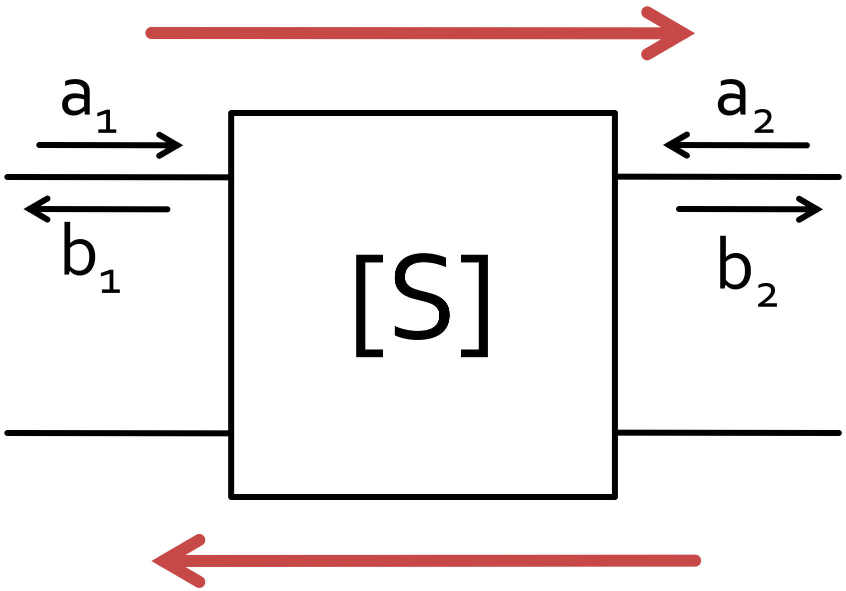


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude **gain** when the other port is **matched**

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

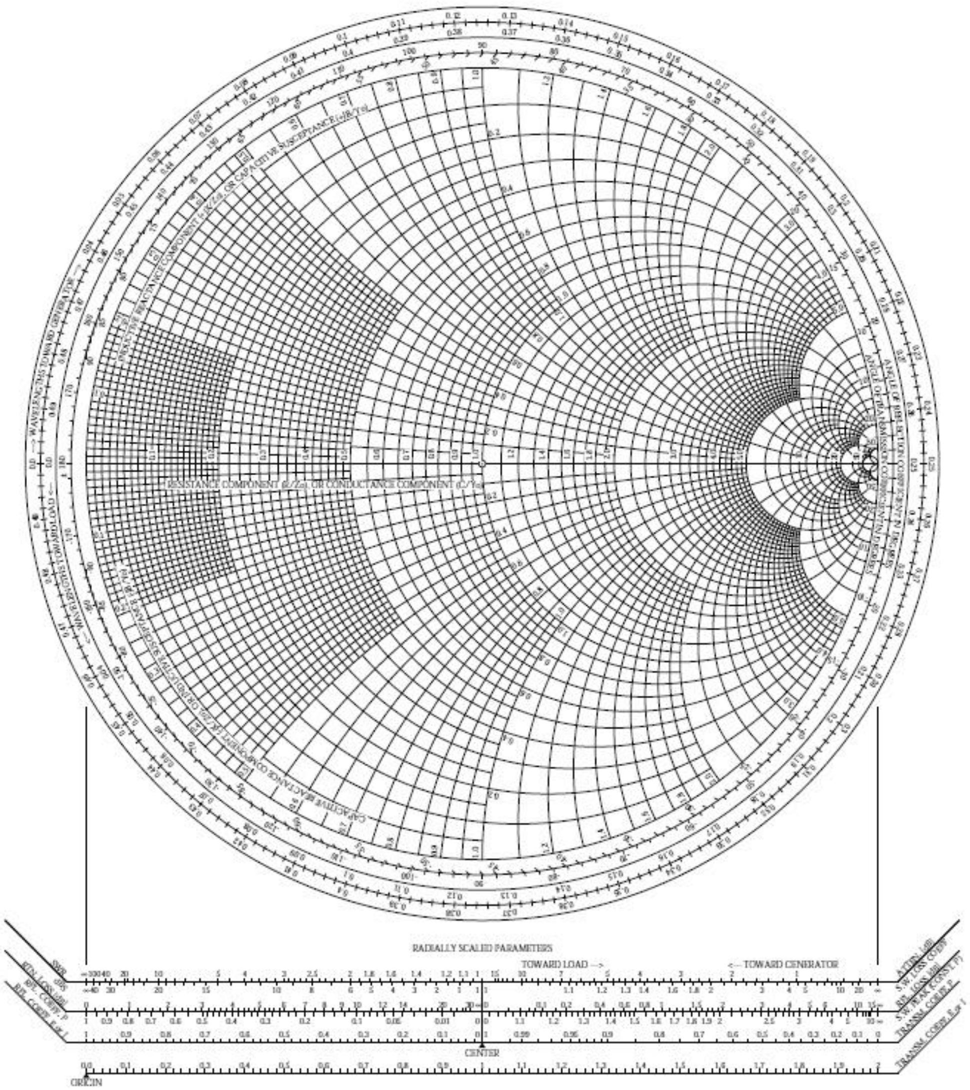
Impedance Matching

The Smith Chart

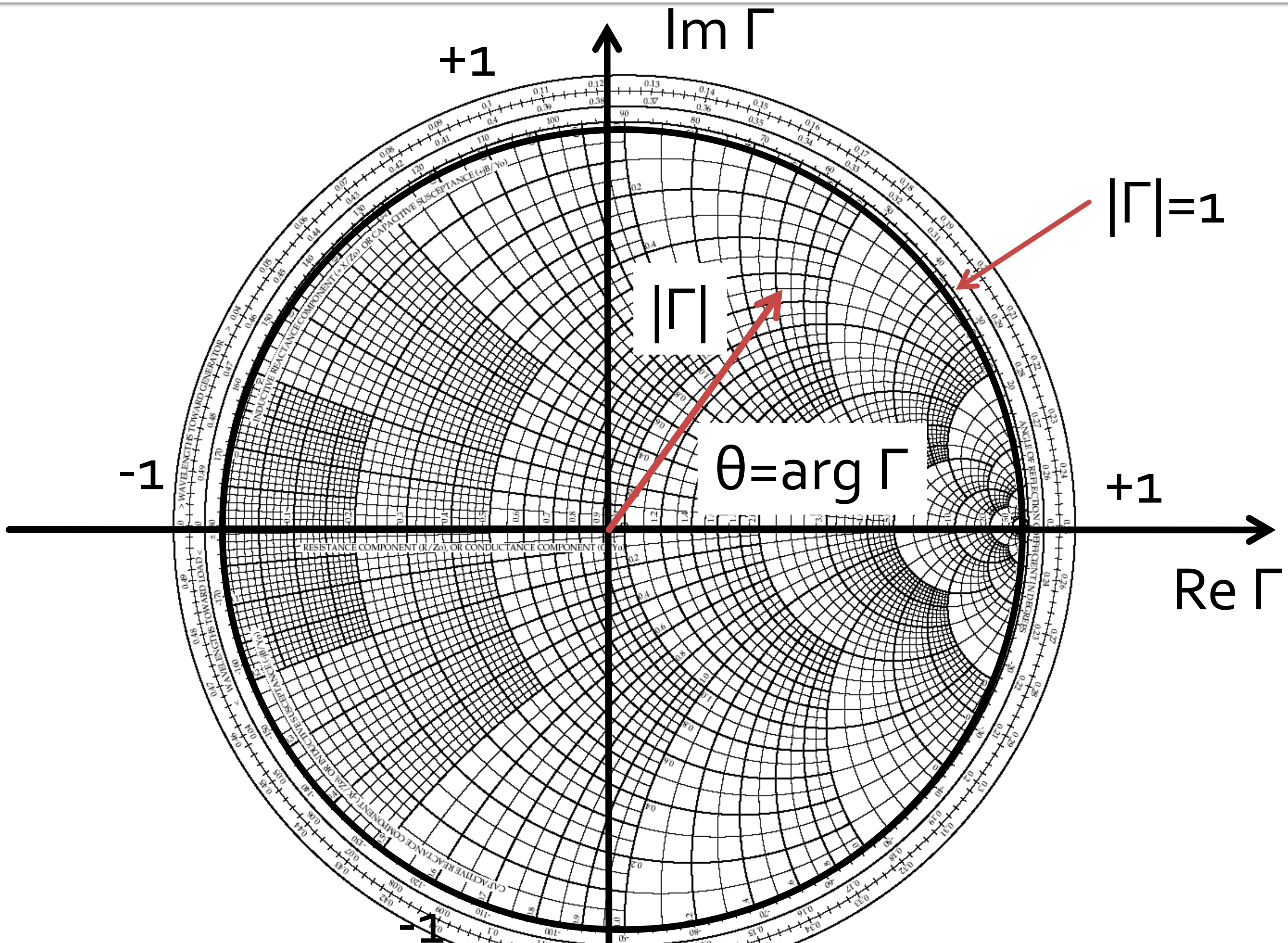
Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

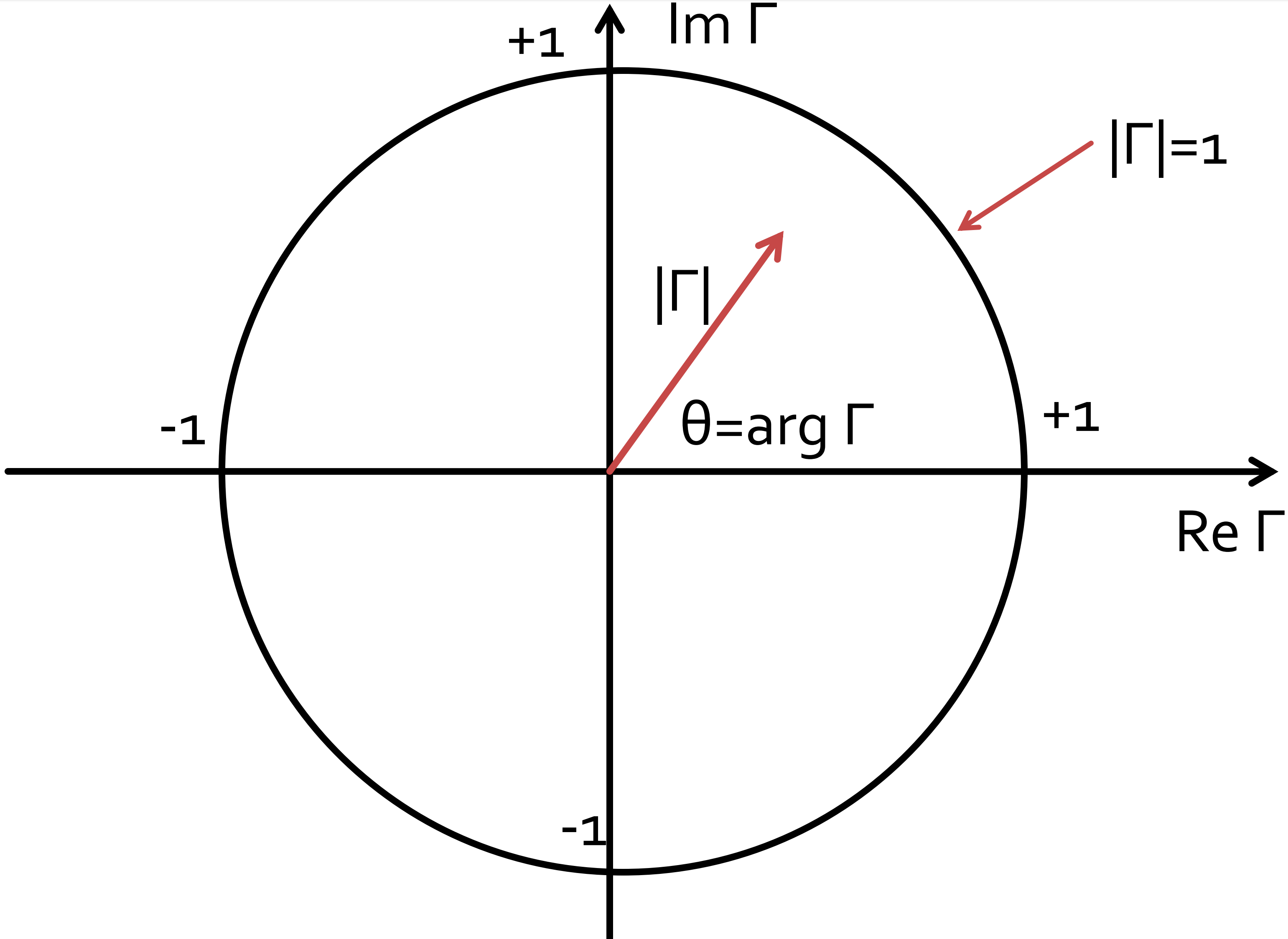
The Smith Chart



The Smith Chart

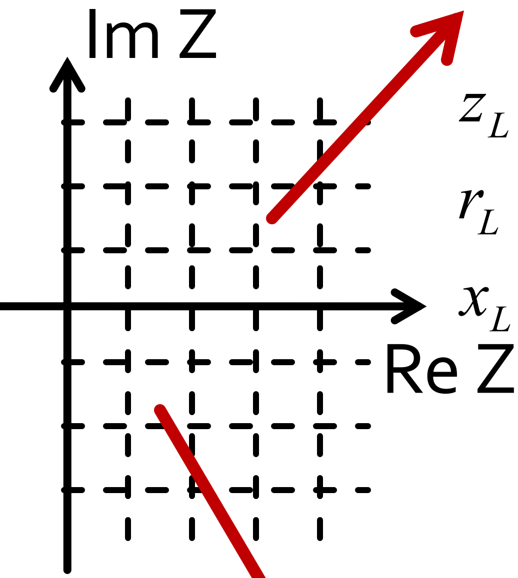


The Smith Chart



The Smith Chart

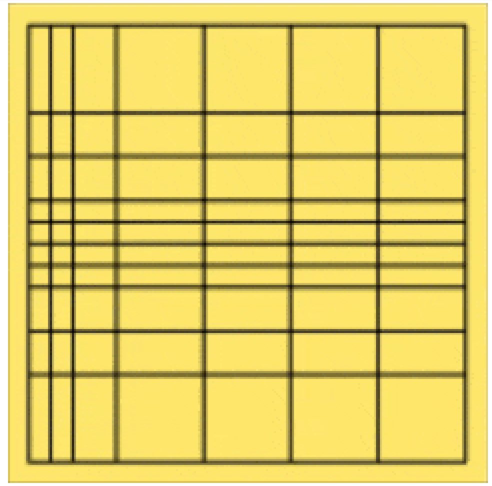
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

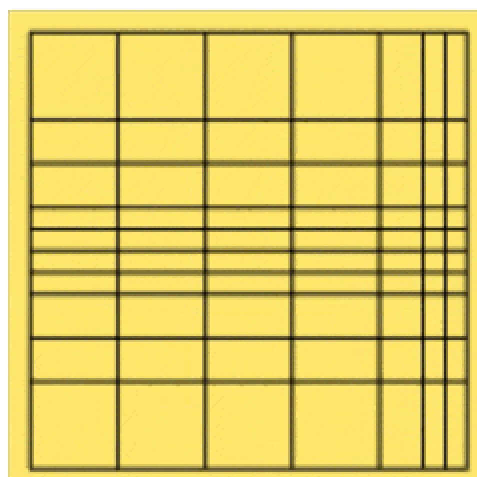


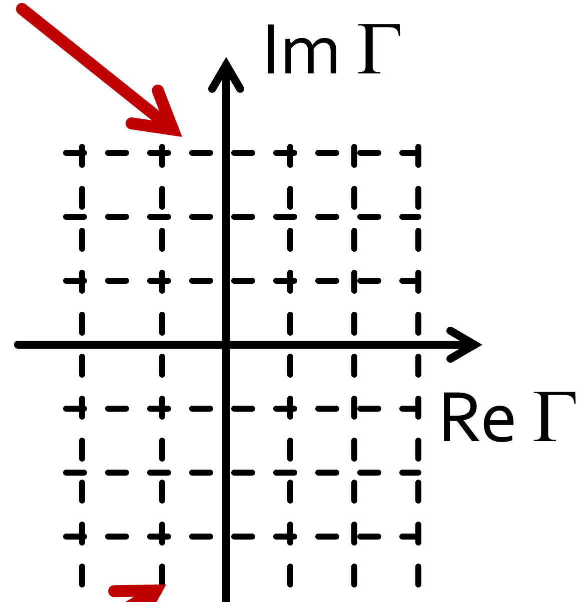
$$z_L = r_L + j \cdot x_L$$

$$r_L = ct.$$

$$x_L = ct.$$

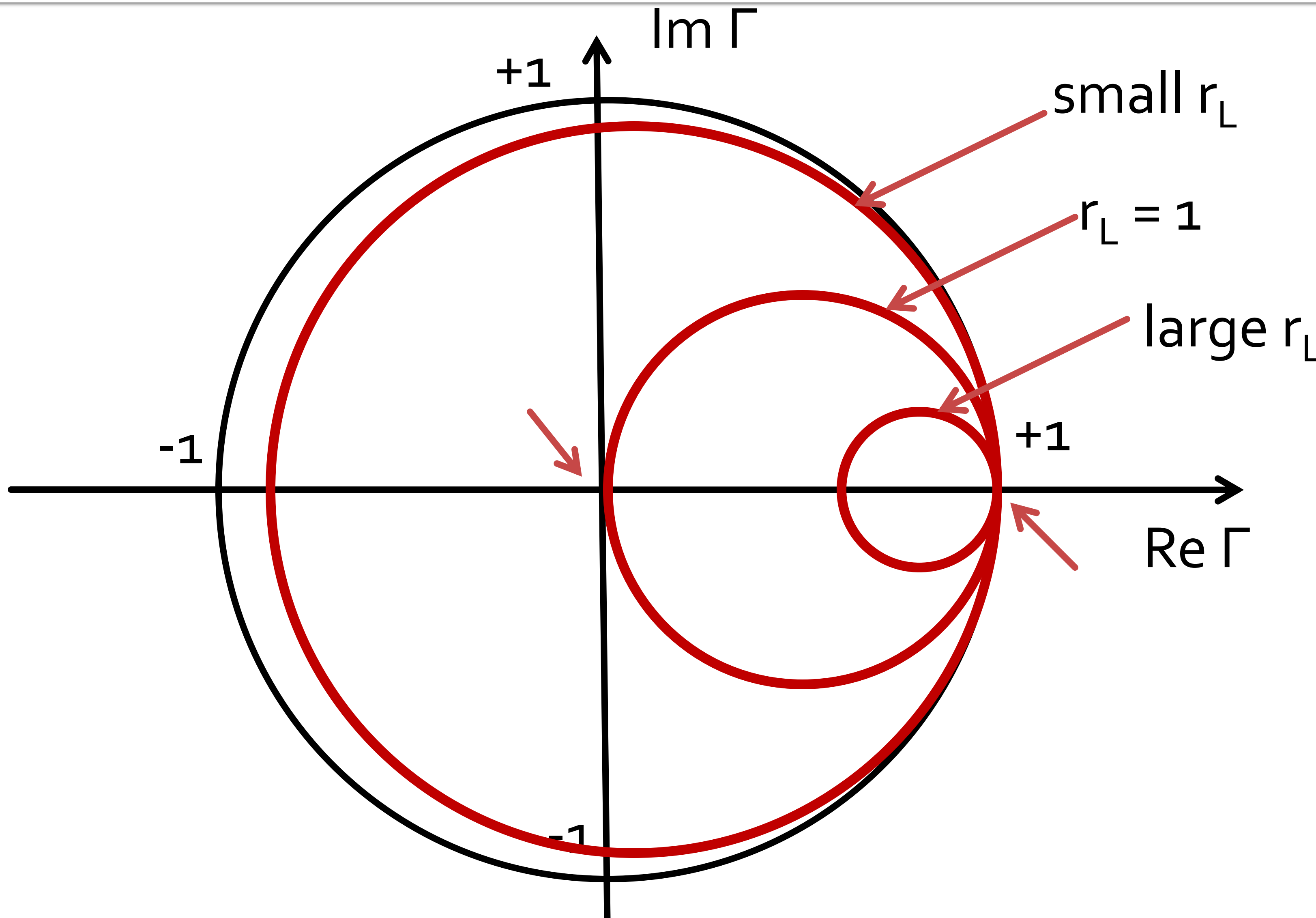




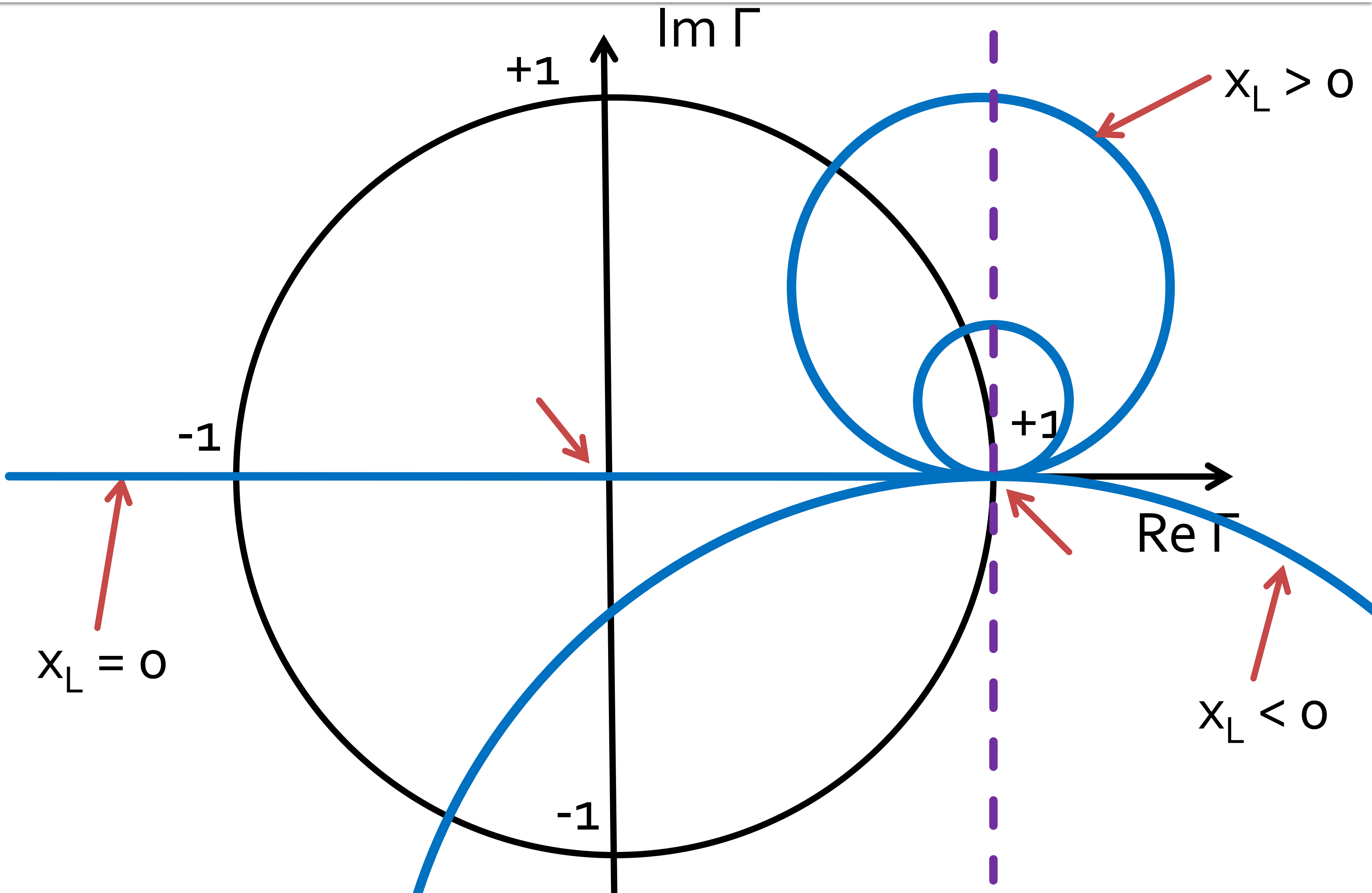


$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$

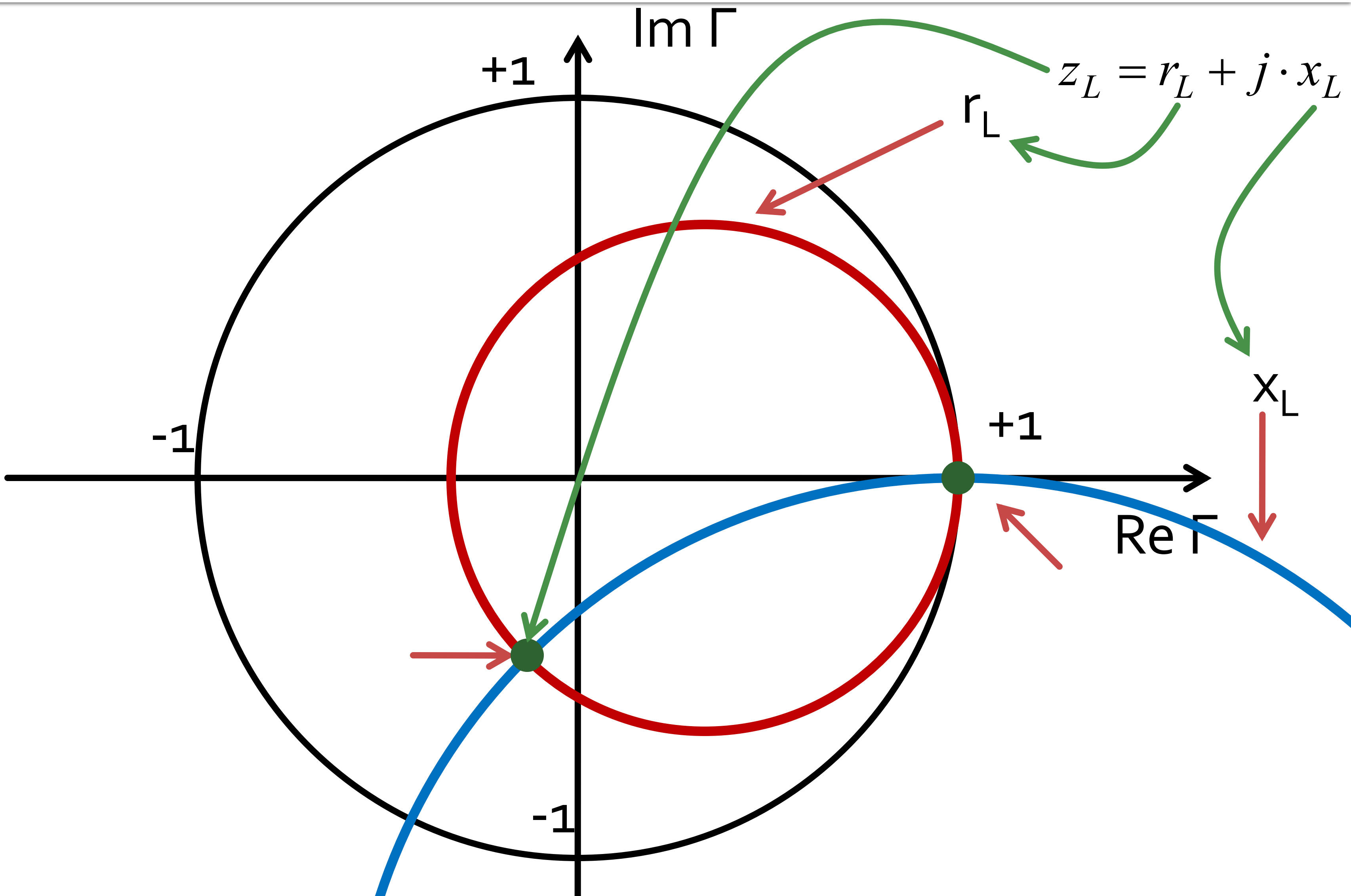
The Smith Chart, resistance



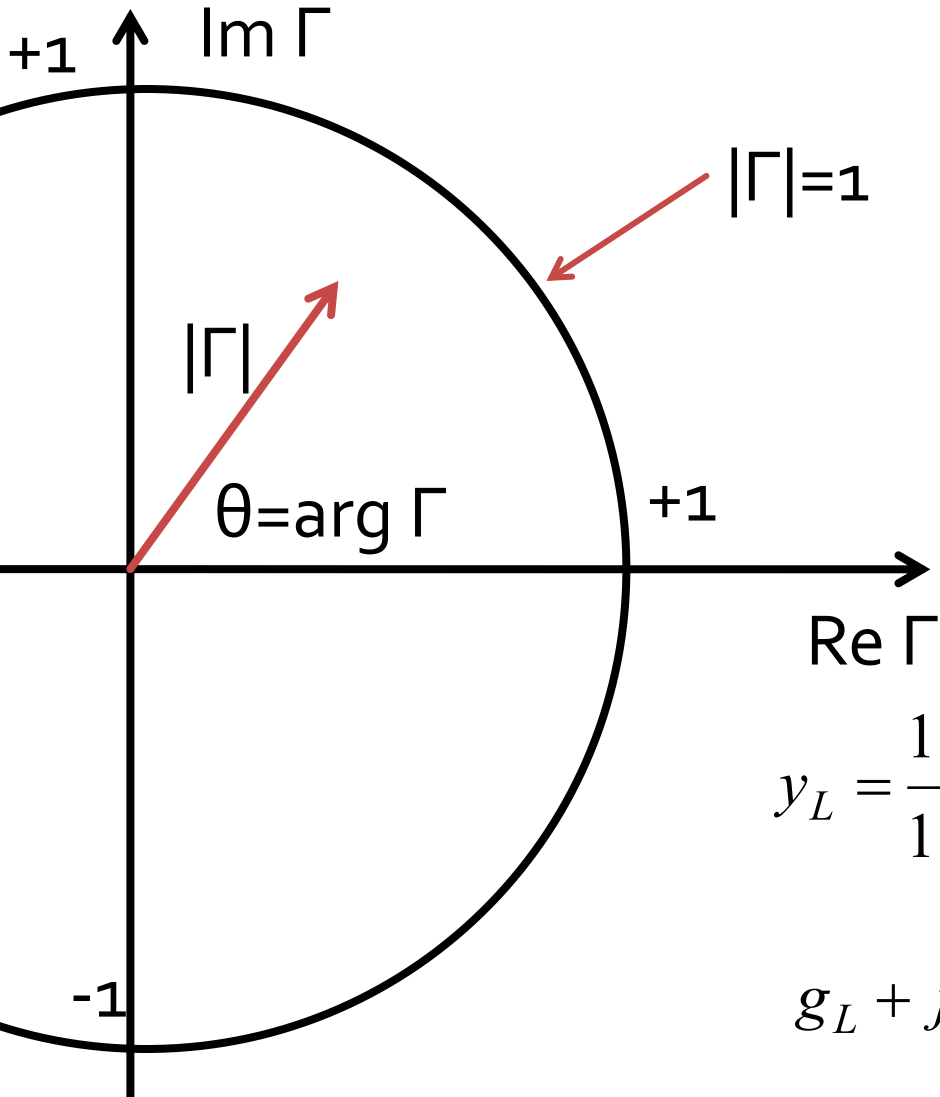
The Smith Chart, reactance



The Smith Chart, impedance



The Admittance Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

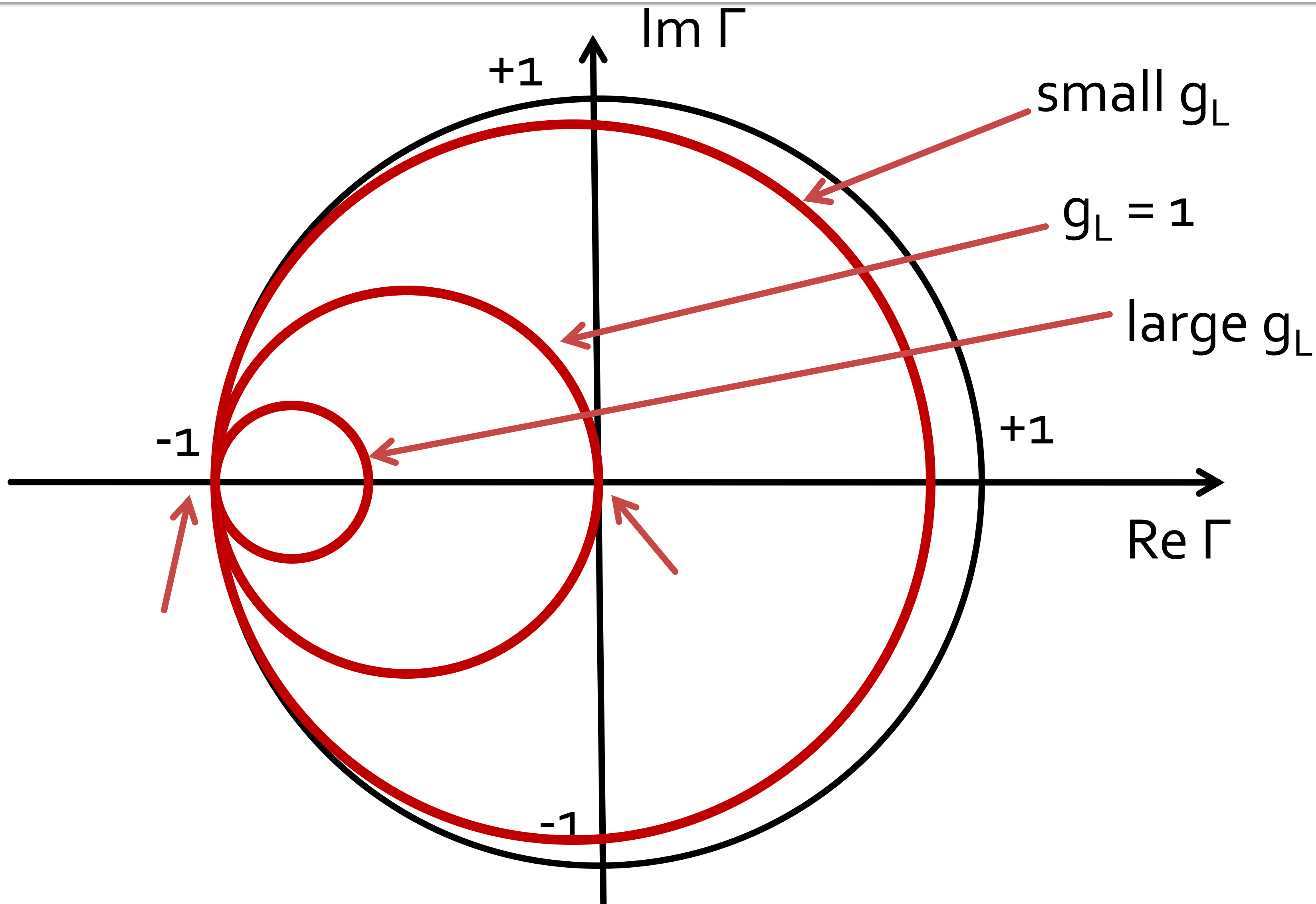
$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

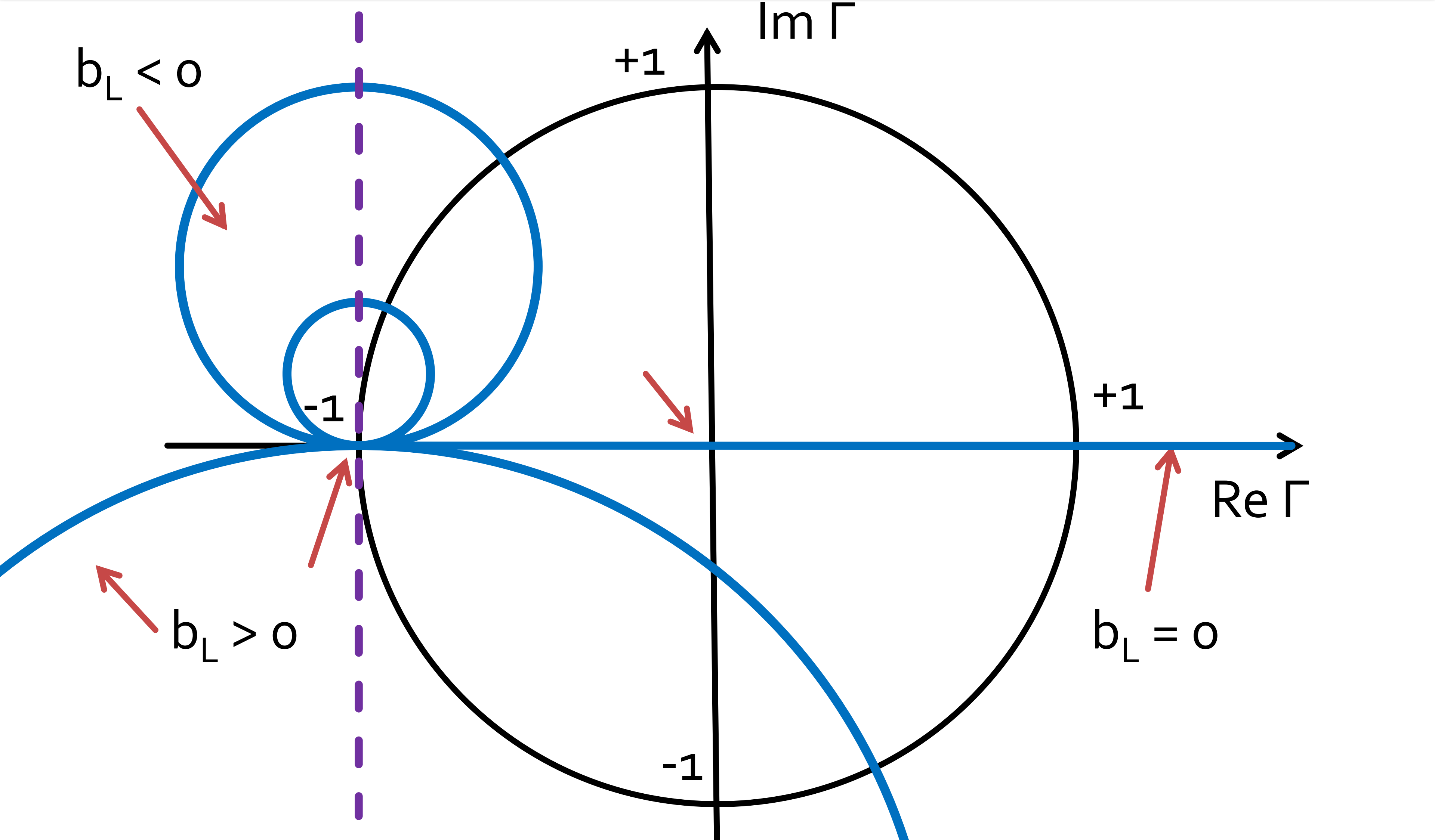
$$y_L = \frac{1 - |\Gamma| \cdot e^{j\theta}}{1 + |\Gamma| \cdot e^{j\theta}} = \frac{1}{r_L + j \cdot x_L} = g_L + j \cdot b_L$$

$$g_L + j \cdot b_L = \frac{(1 - \Gamma_r) - j \cdot \Gamma_i}{(1 + \Gamma_r) + j \cdot \Gamma_i}$$

The Smith Chart, conductance



The Smith Chart, susceptance



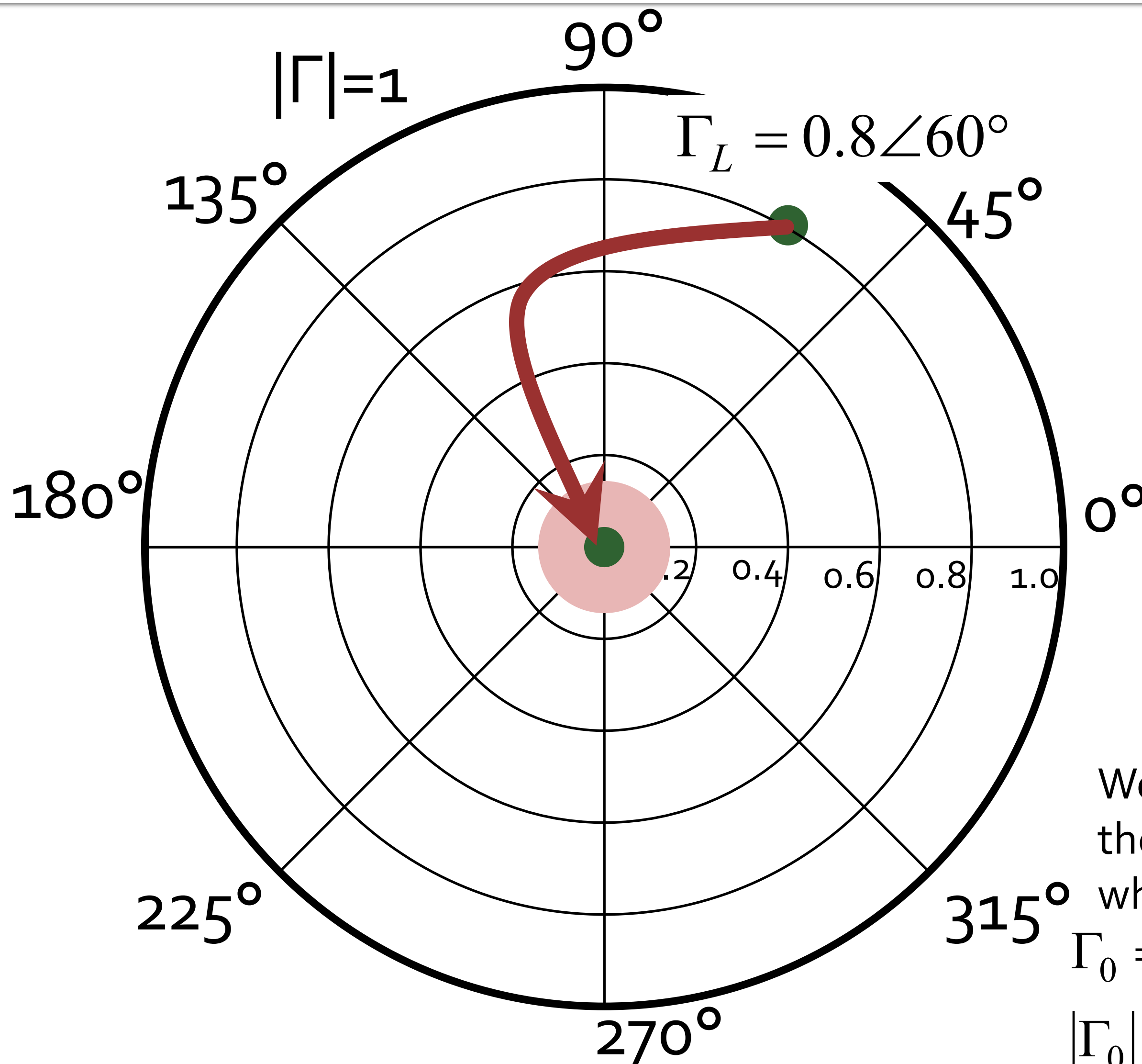
Impedance matching

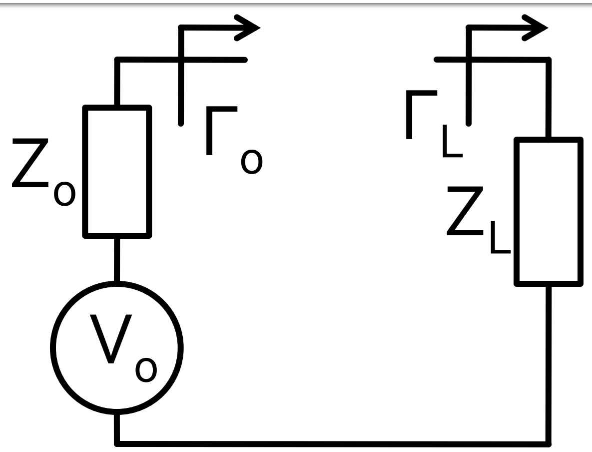
Impedance Matching with lumped elements (L Networks)

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The Smith Chart, reflection coefficient, impedance matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

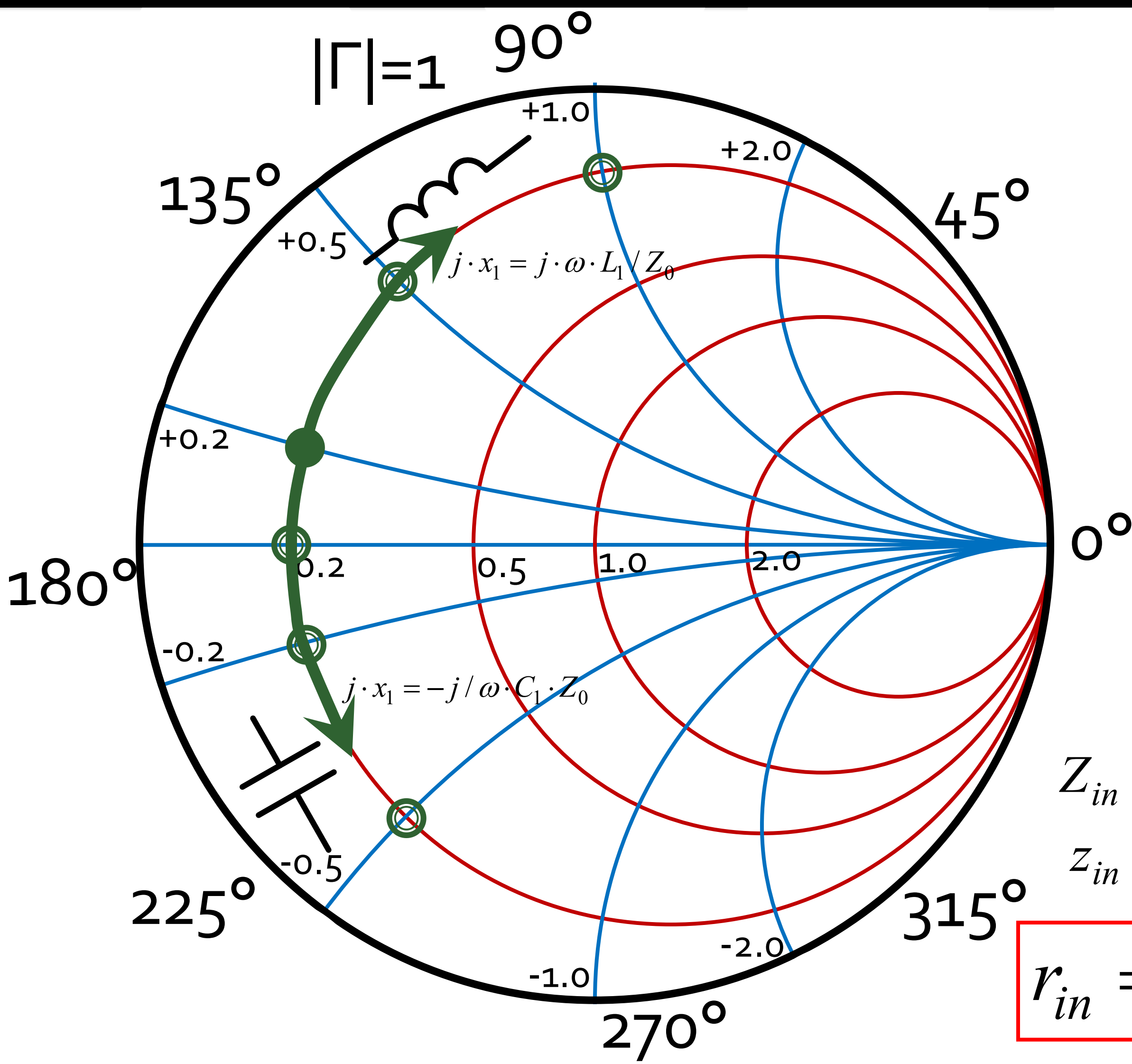
$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$
$$z_L = 0.429 + j \cdot 1.65$$
$$\Gamma_L = 0.8\angle 60°$$

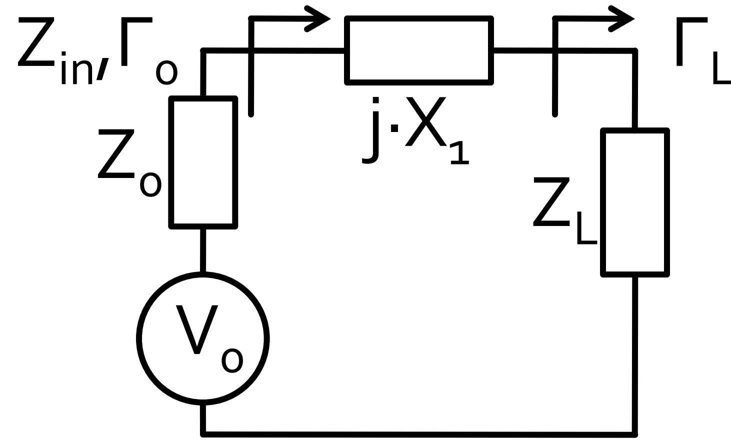
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ "good enough" match

The Smith Chart, series reactance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

$$Z_{in} = Z_L + j\cdot X_1 = R_L + j\cdot\left(X_L + X_1\right)$$

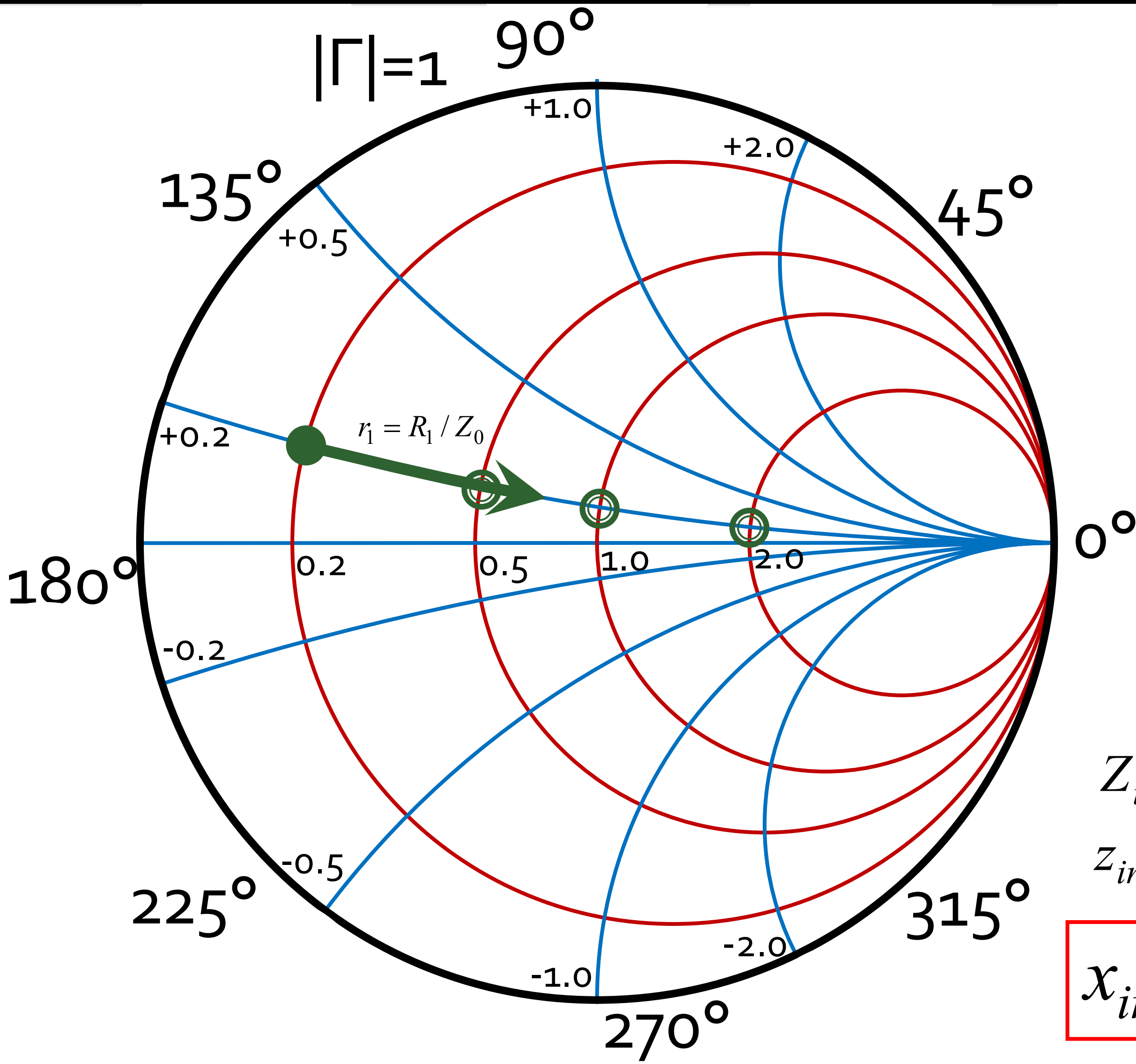
$$z_{in} = r_L + j\cdot\left(x_L + x_1\right)$$

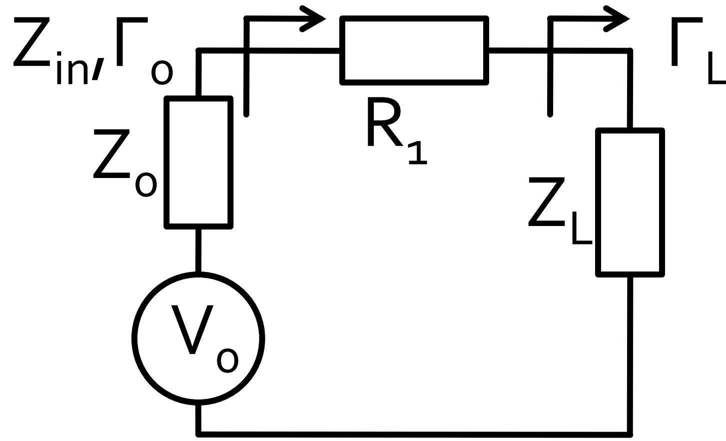
$$r_{in} = r_L$$

$$j\cdot x_1 = j\cdot\omega\cdot L_1/Z_0 > 0$$

$$j\cdot x_1 = -j/\omega\cdot C_1\cdot Z_0 < 0$$

The Smith Chart, series resistance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

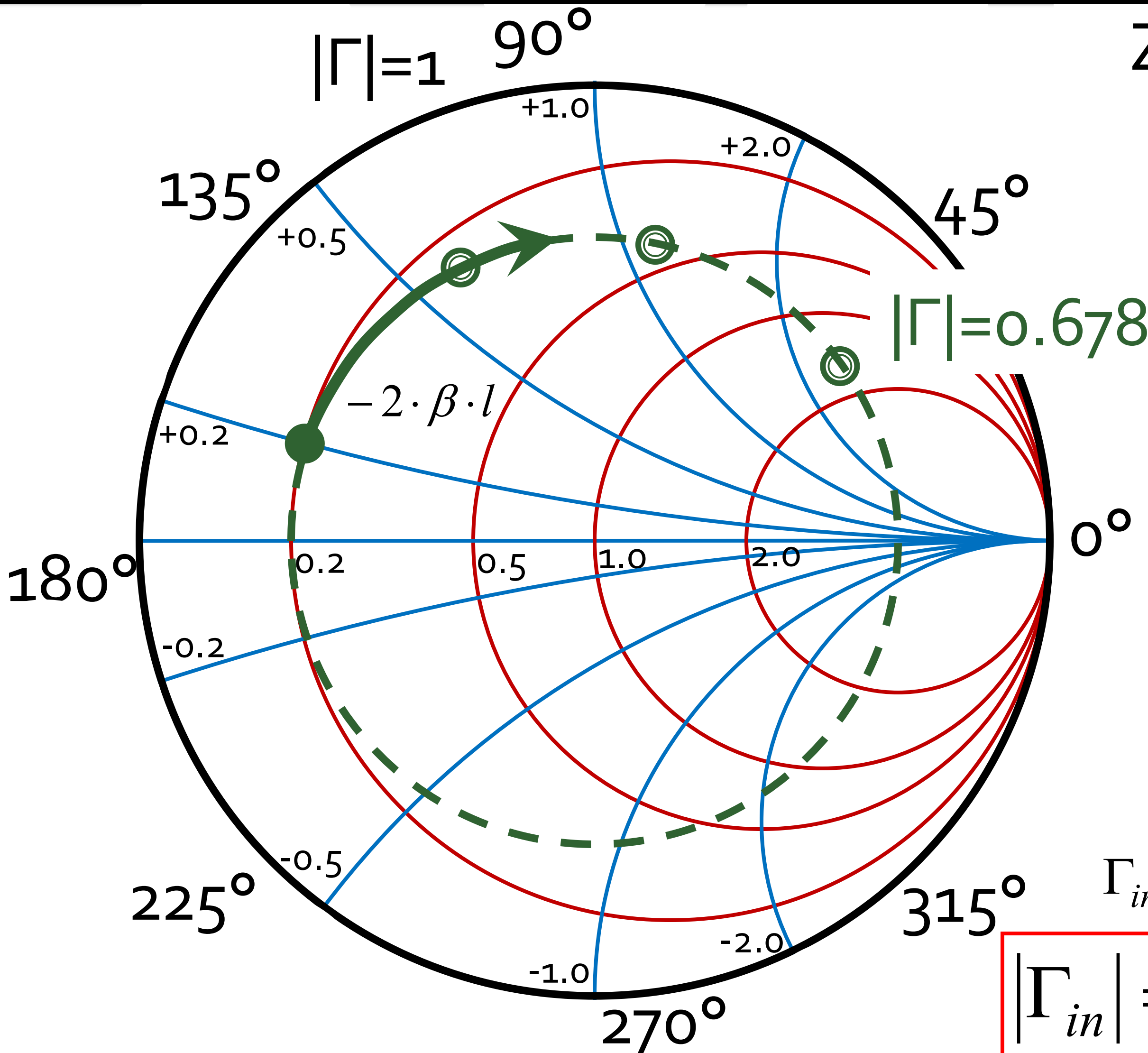
$$Z_{in} = Z_L + R_1 = (R_L + R_1) + j \cdot X_L$$

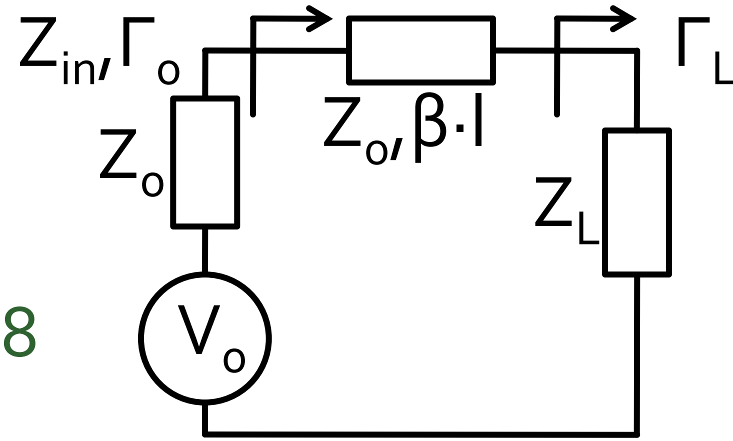
$$z_{in} = z_L + r_1 = (r_L + r_1) + j \cdot x_L$$

$$x_{in} = x_L$$

$$r_{in} = r_L + R_1 / Z_0$$

The Smith Chart, series transmission line, Z_o





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10\Omega$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

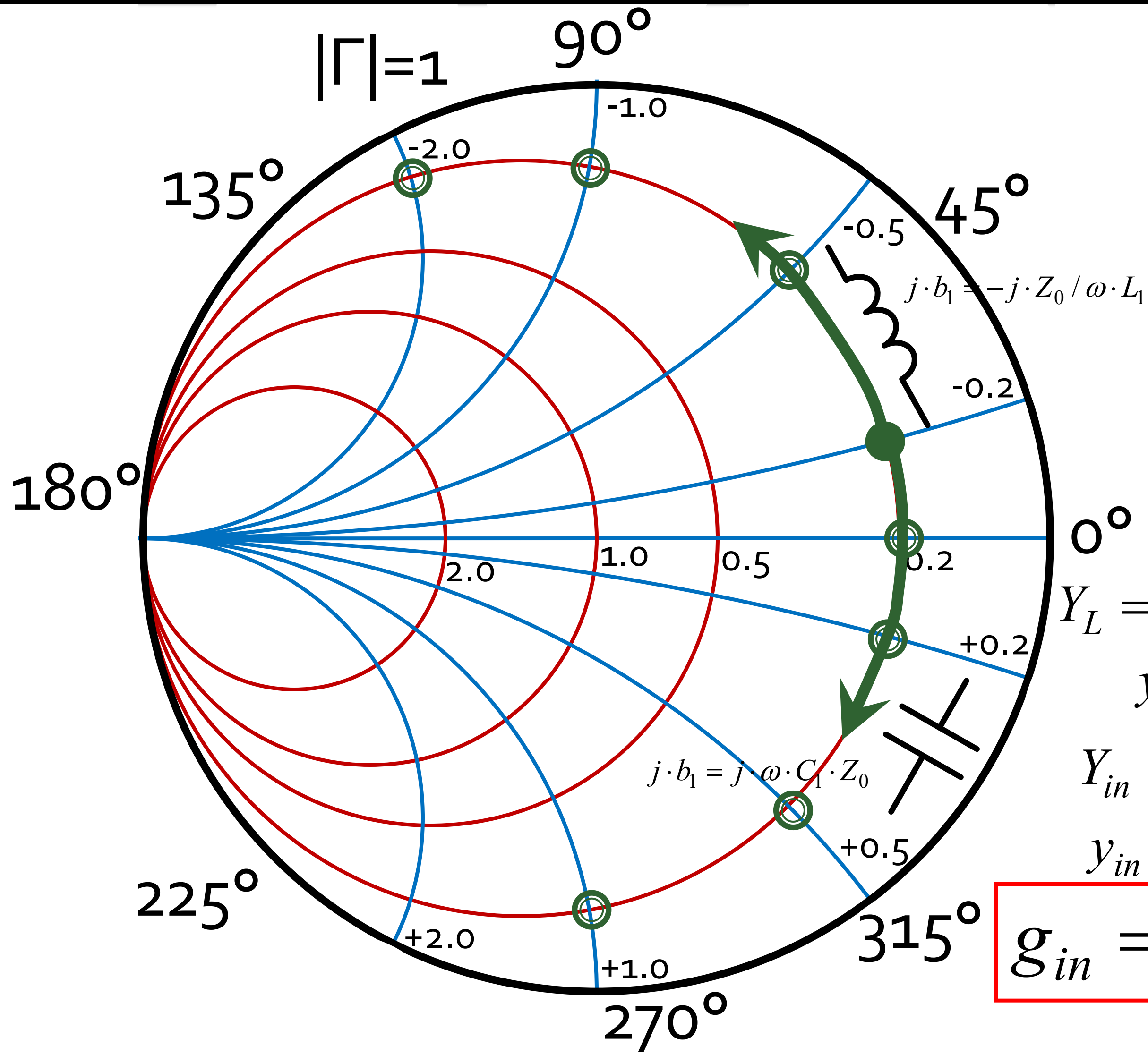
$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}$$

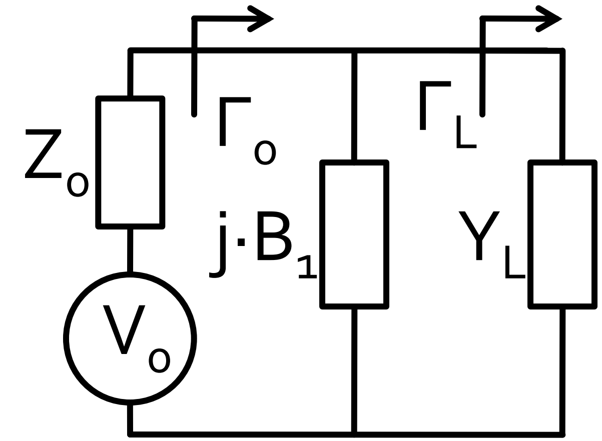
$$\Gamma_{in} = \Gamma_L\cdot e^{-2j\cdot\beta\cdot l}$$

$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2\cdot\beta\cdot l$$

The Smith Chart, shunt susceptance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

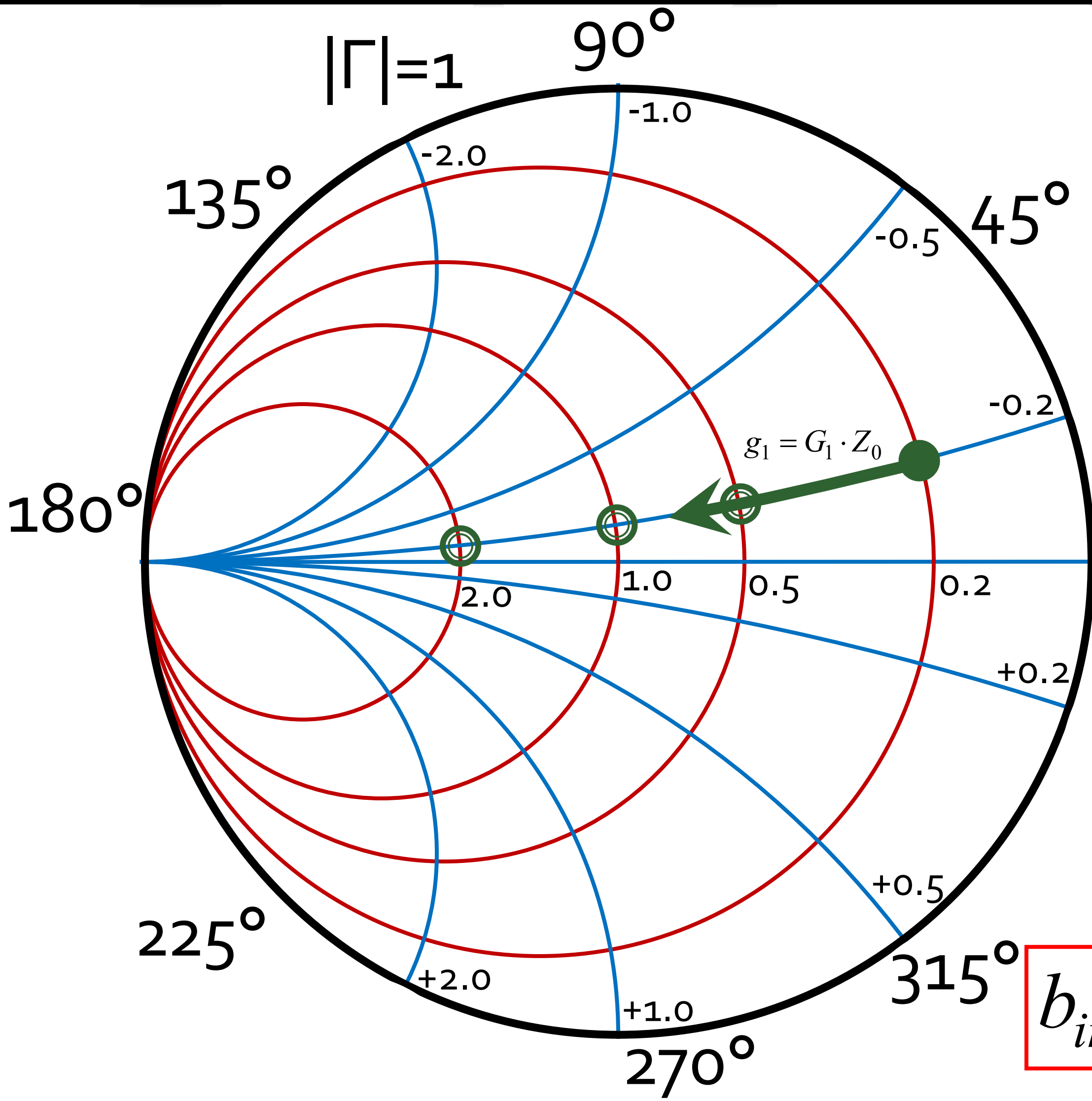
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

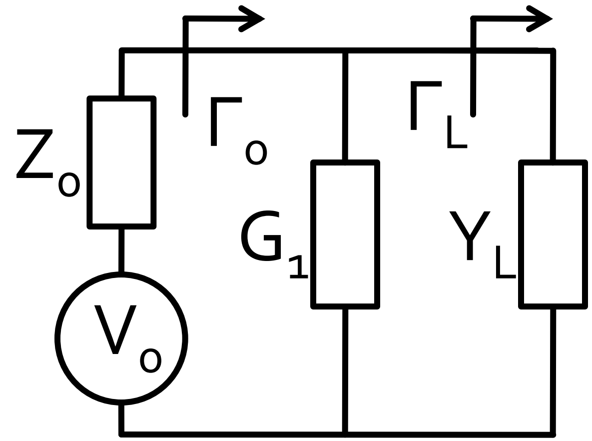
$$g_{in} = g_L$$

$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

The Smith Chart, shunt conductance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

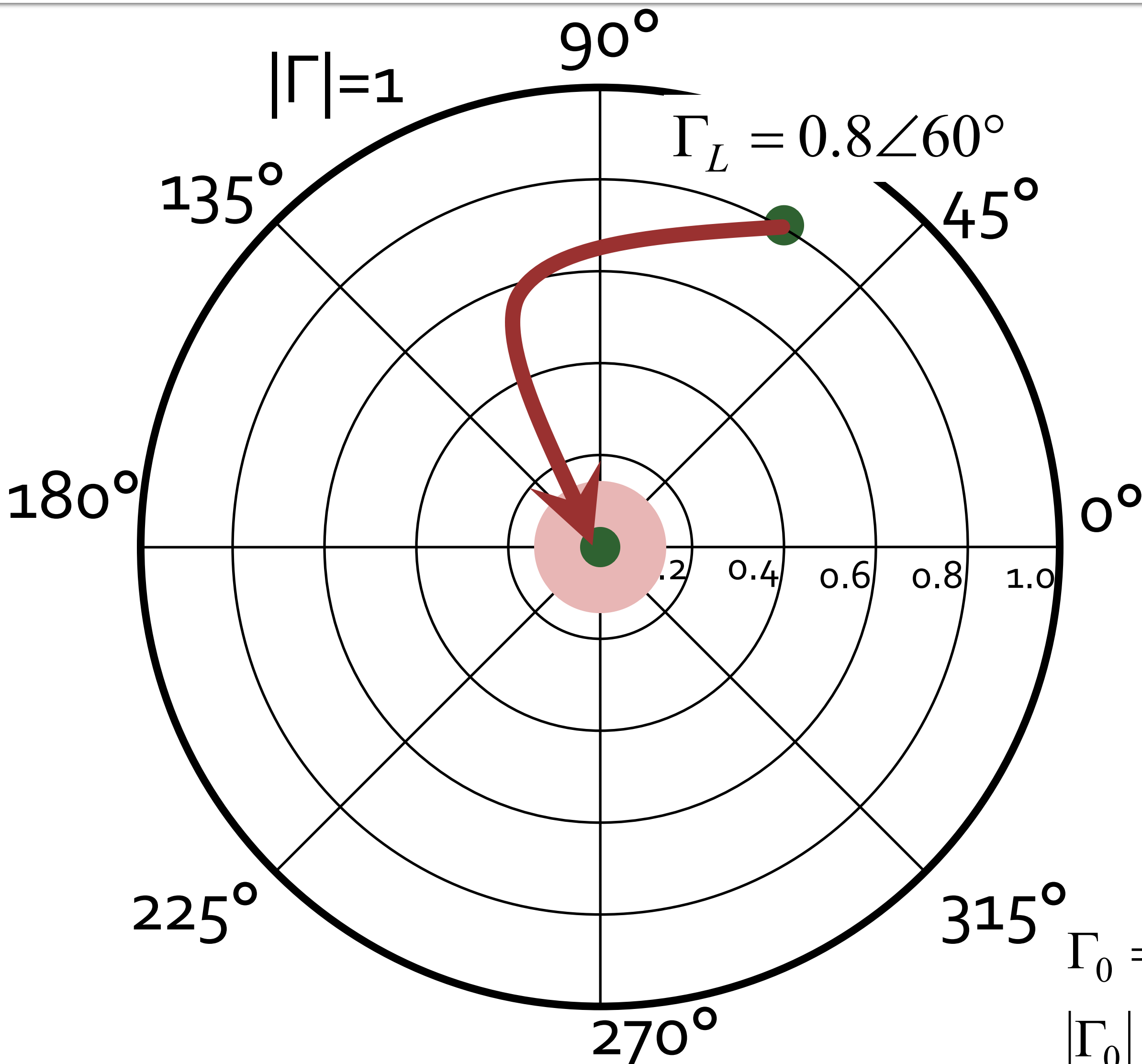
$$Y_{in} = Y_L + G_1 = (G_L + G_1) + j \cdot B_L$$

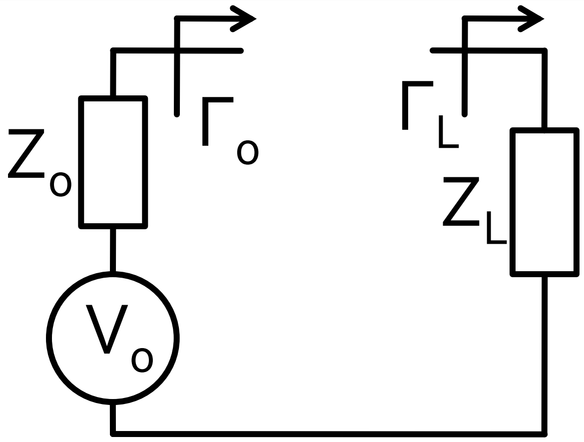
$$y_{in} = (g_L + g_1) + j \cdot b_L$$

$$b_{in} = b_L$$

$$g_{in} = g_L + G_1 \cdot Z_0$$

Impedance matching



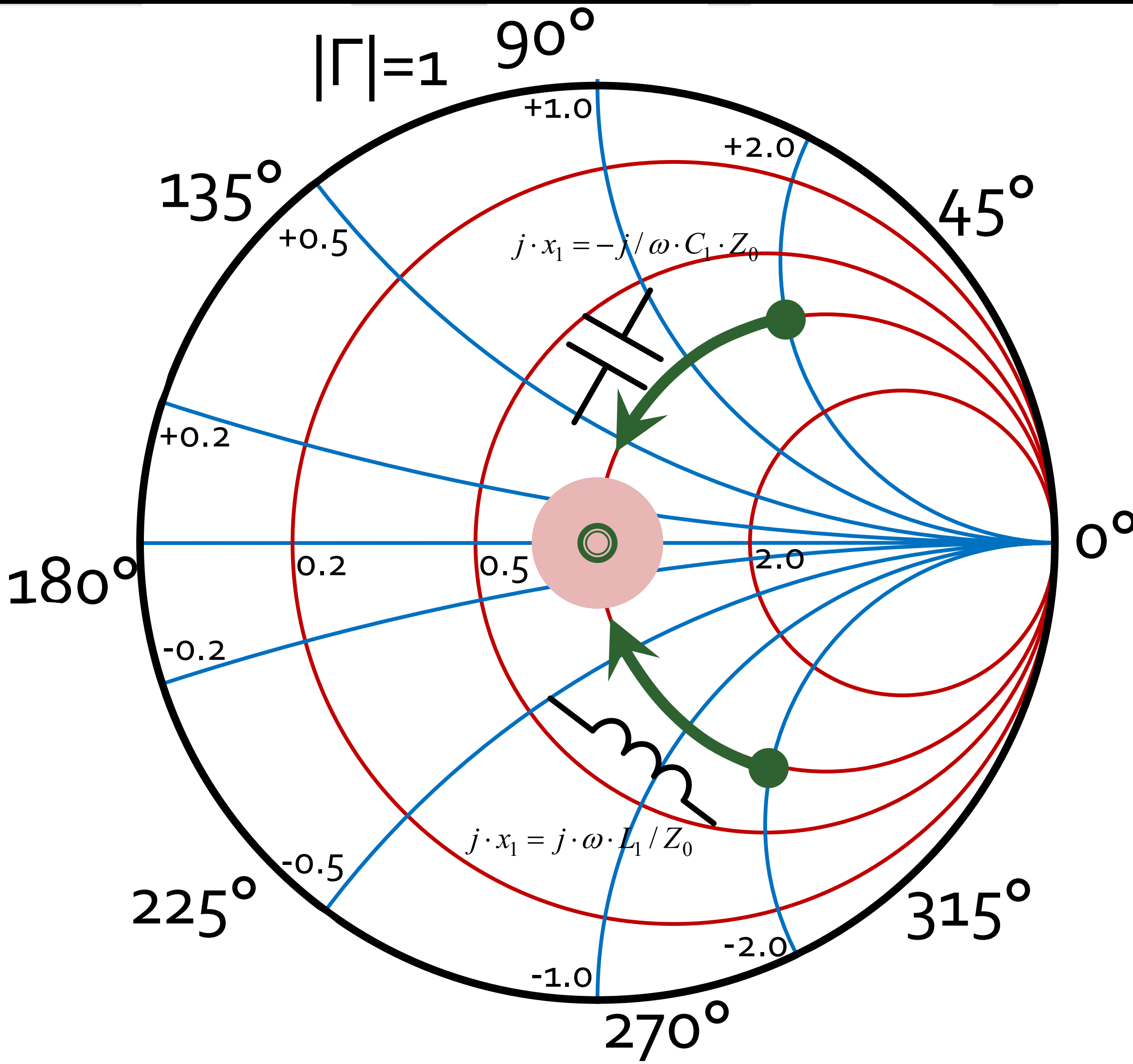


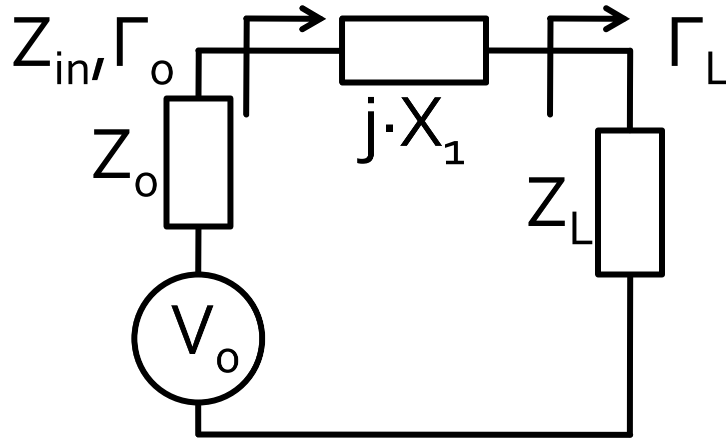
How?

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

Matching, series reactance





$$z_L = r_L + j \cdot x_L$$

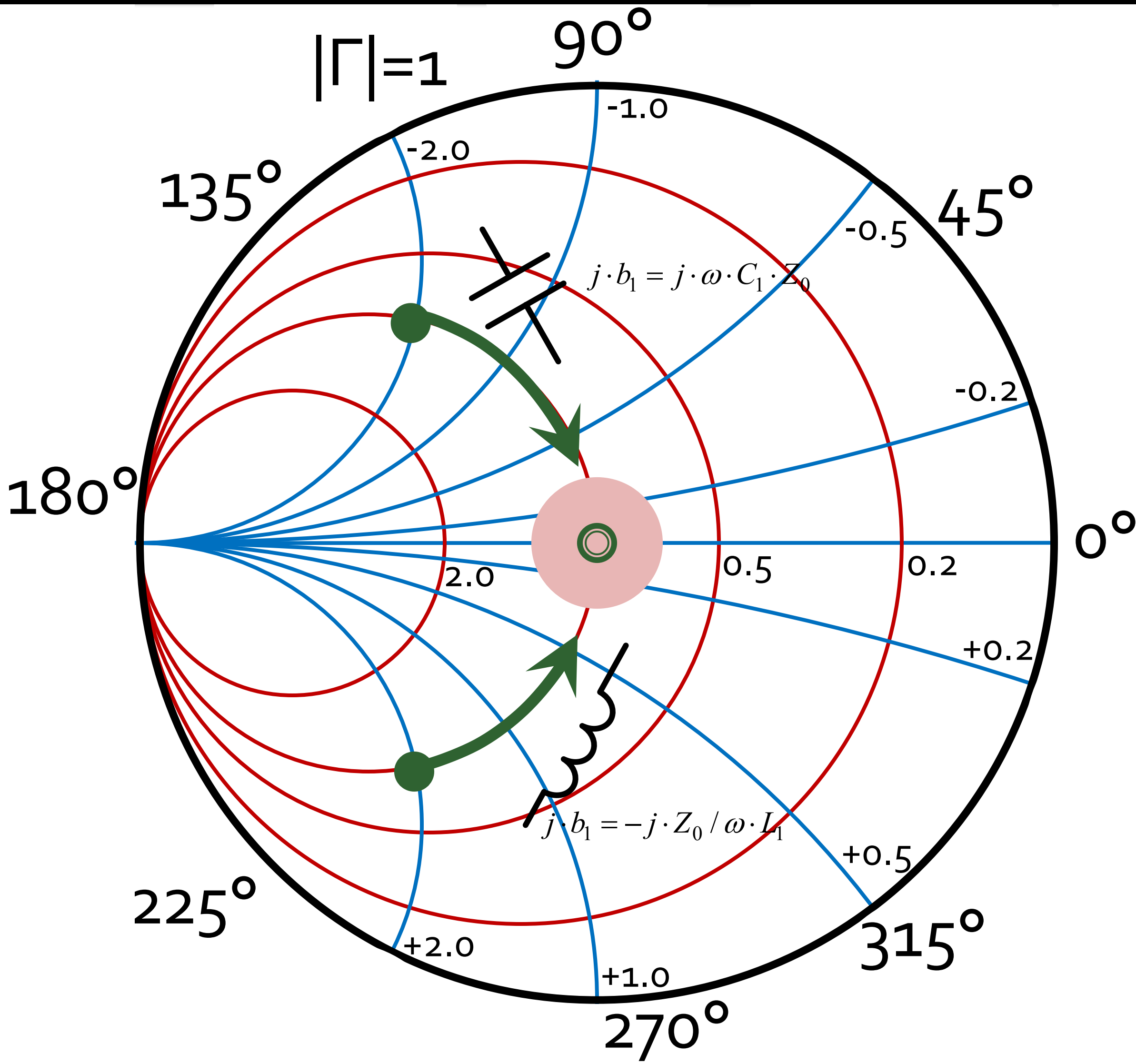
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

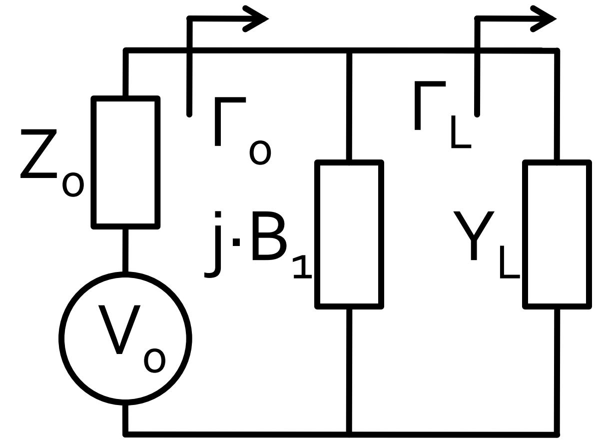
$$r_{in} = r_L$$

- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Matching, shunt susceptance





$$y_L = g_L + j \cdot b_L$$

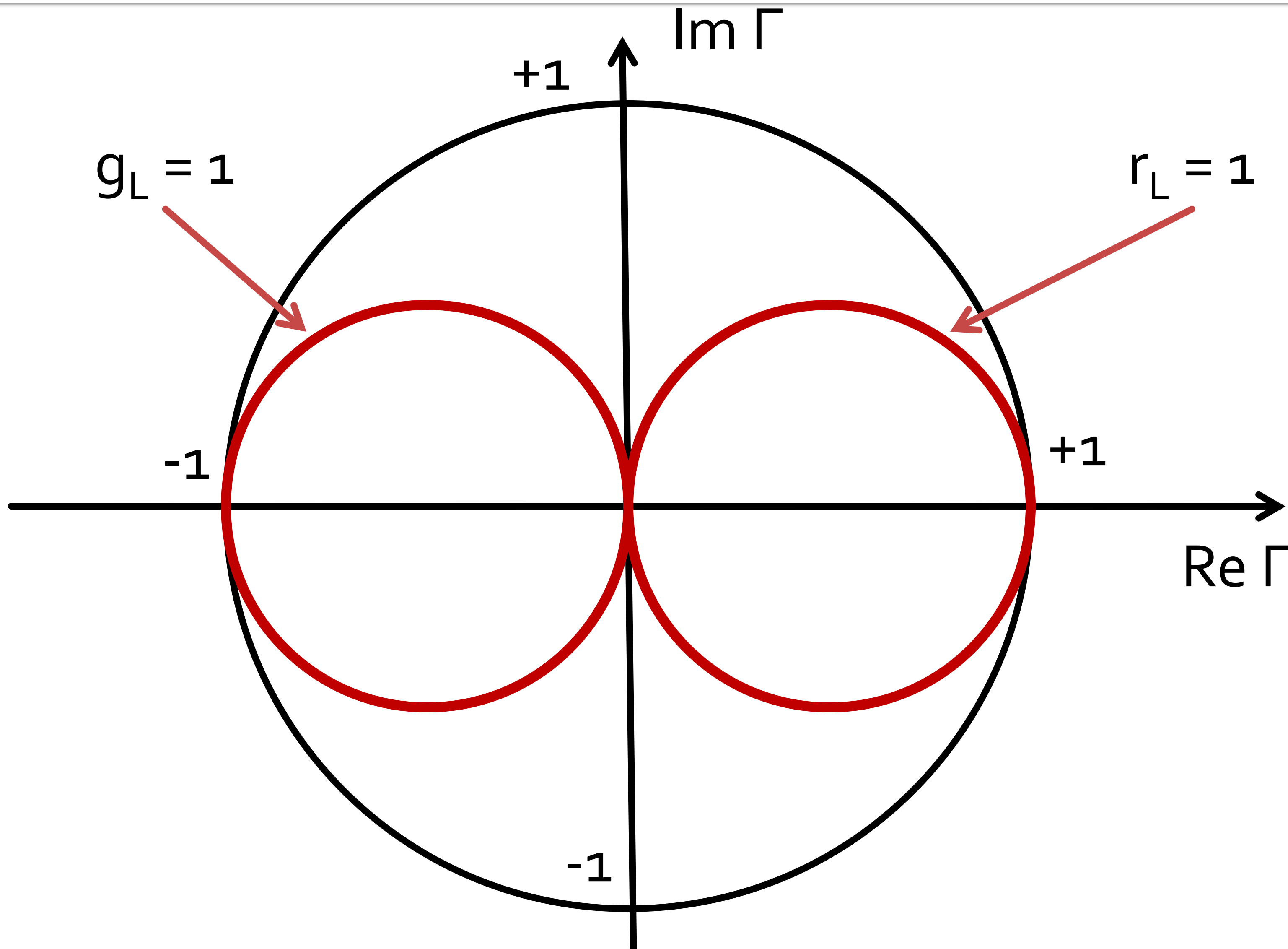
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

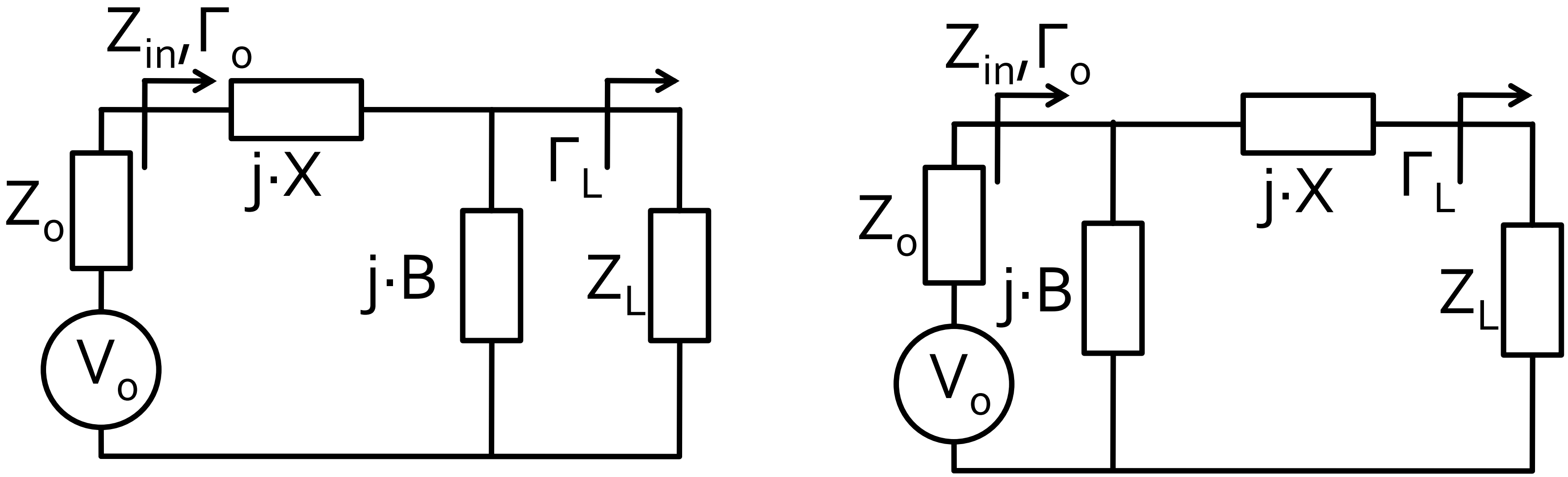
- Match can be obtained **if and only if** $g_L = 1$
- we compensate the reactive part of the load

$$j \cdot b_1 = -j \cdot b_L$$

Smith chart, r=1 and g=1

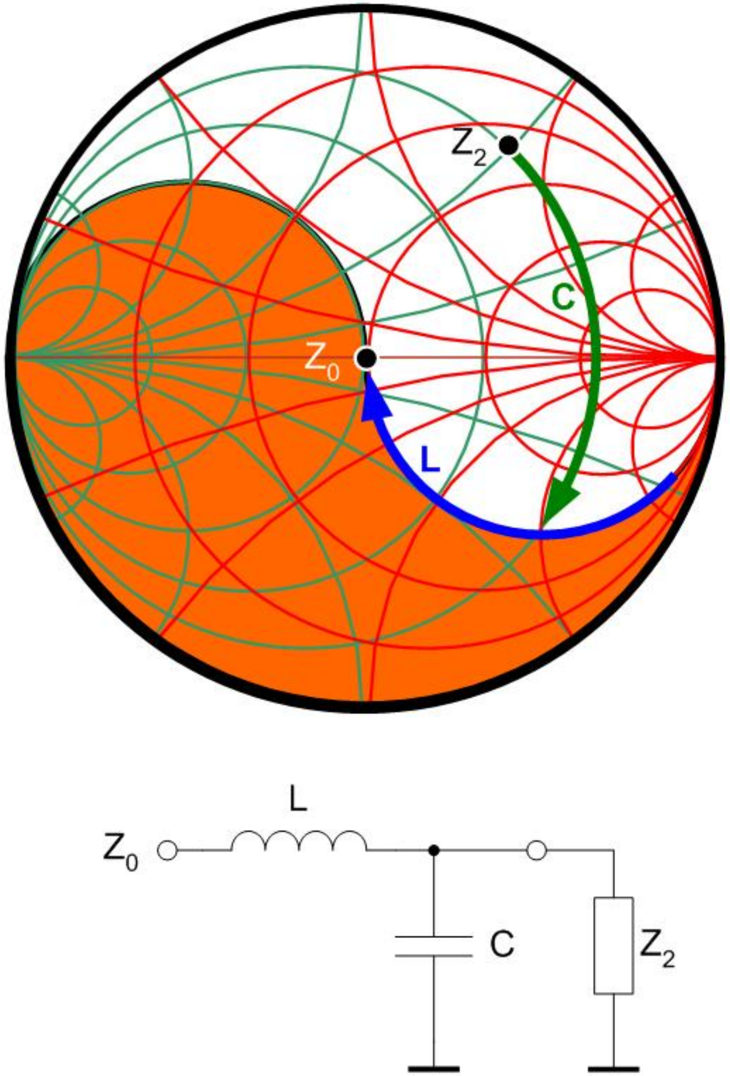


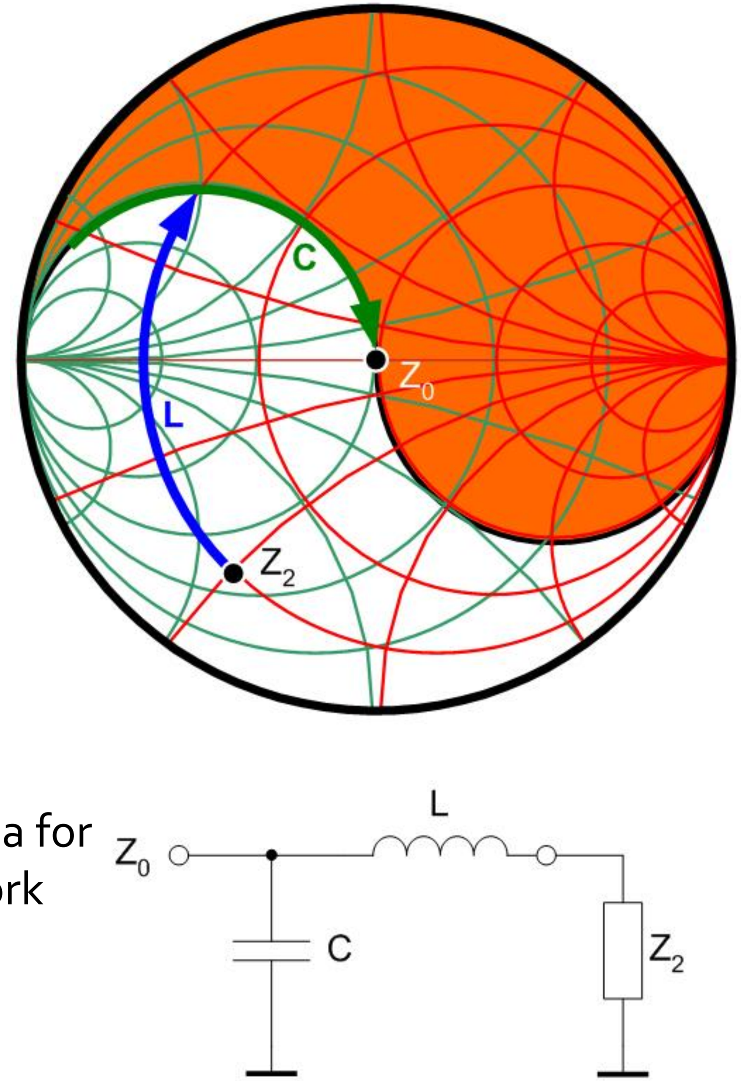
Matching with 2 reactive elements (L Networks)



- Two steps matching
  - first reactive element moves the reflection coefficient **on the circle** $r_L = 1/g_L = 1$
  - second element compensates the remaining reactance and achieves the impedance match

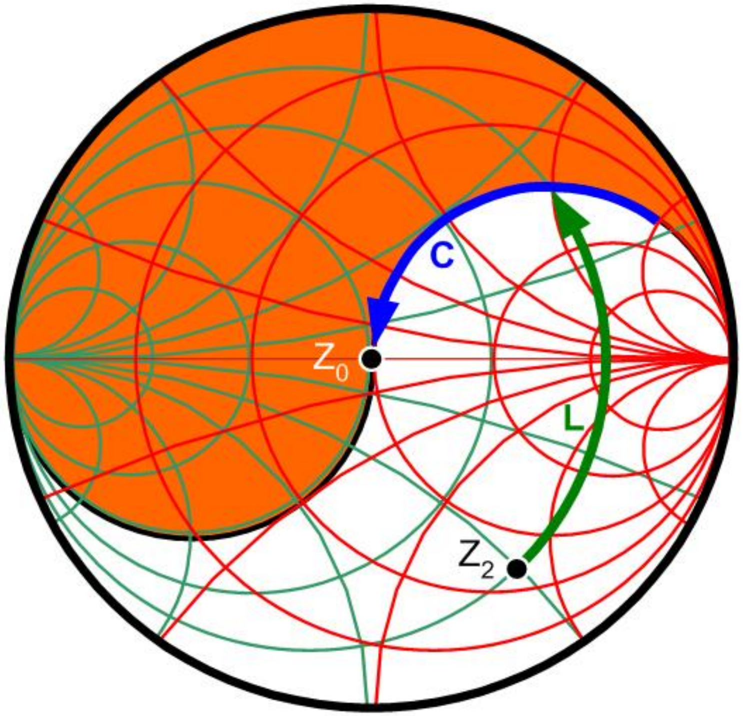
series L, shunt C / shunt C, series L

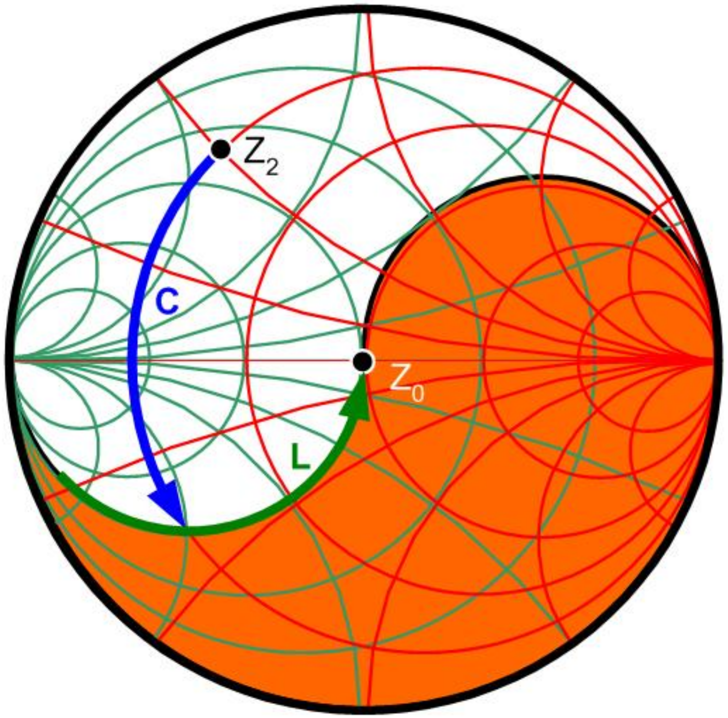


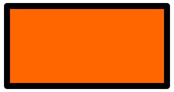


Forbidden area for current network

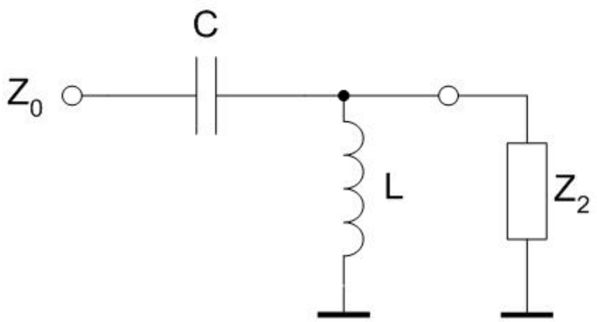
series C, shunt L / shunt L, series C

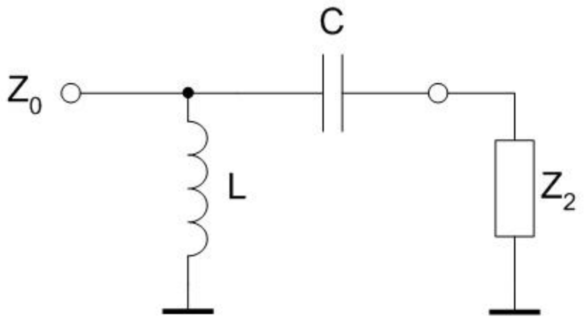




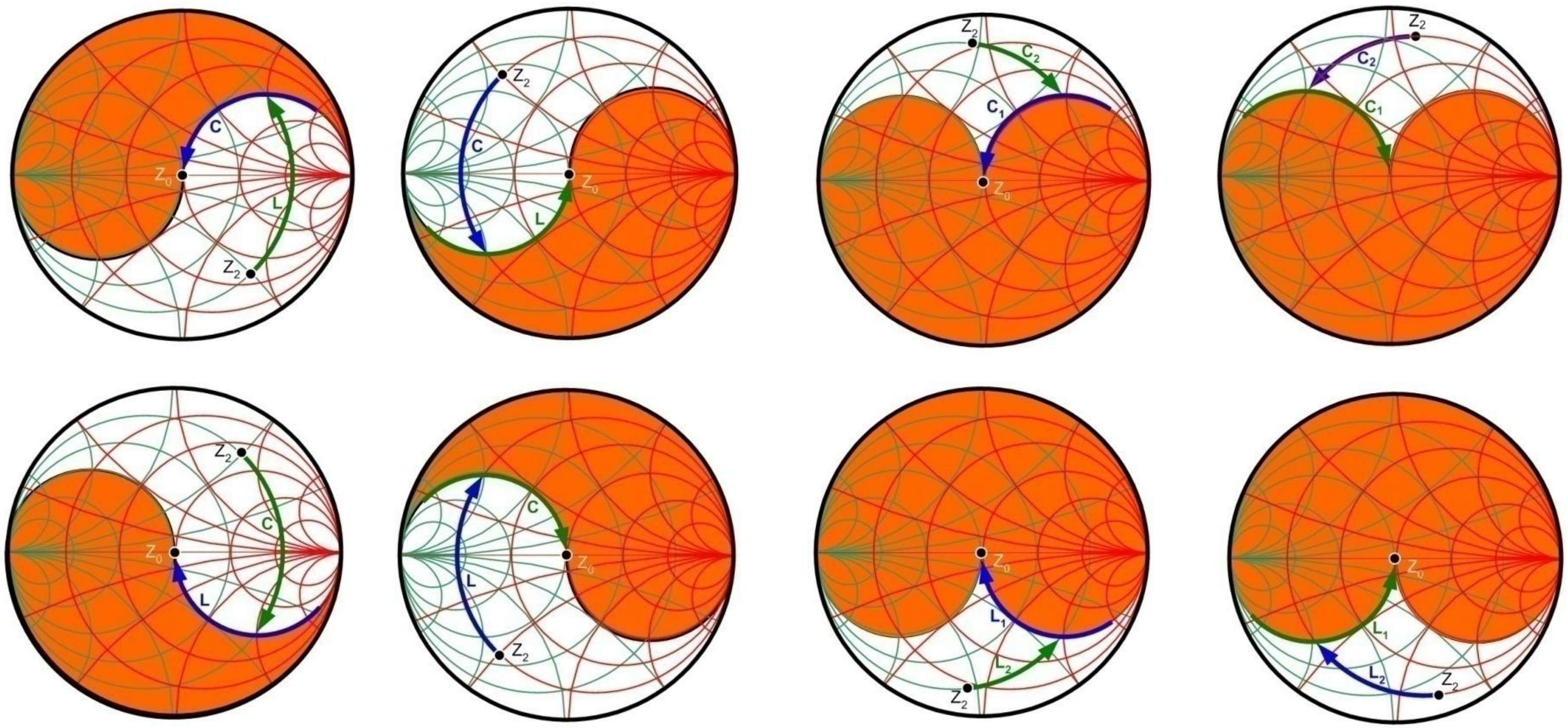


Forbidden area for current network



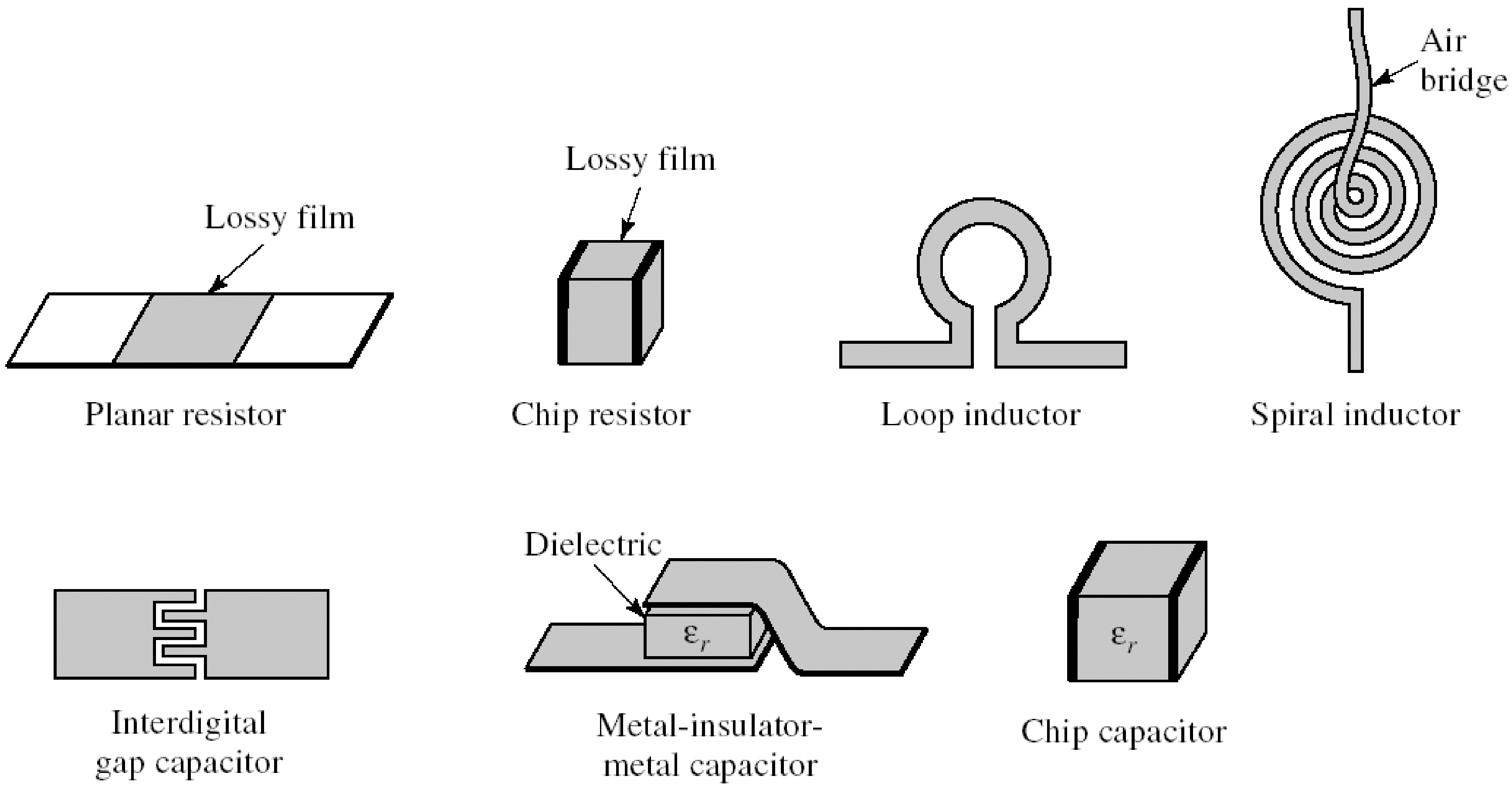


Matching with 2 reactive elements (L Networks)



Forbidden area for current network

Practical realization of lumped elements for microwave frequencies



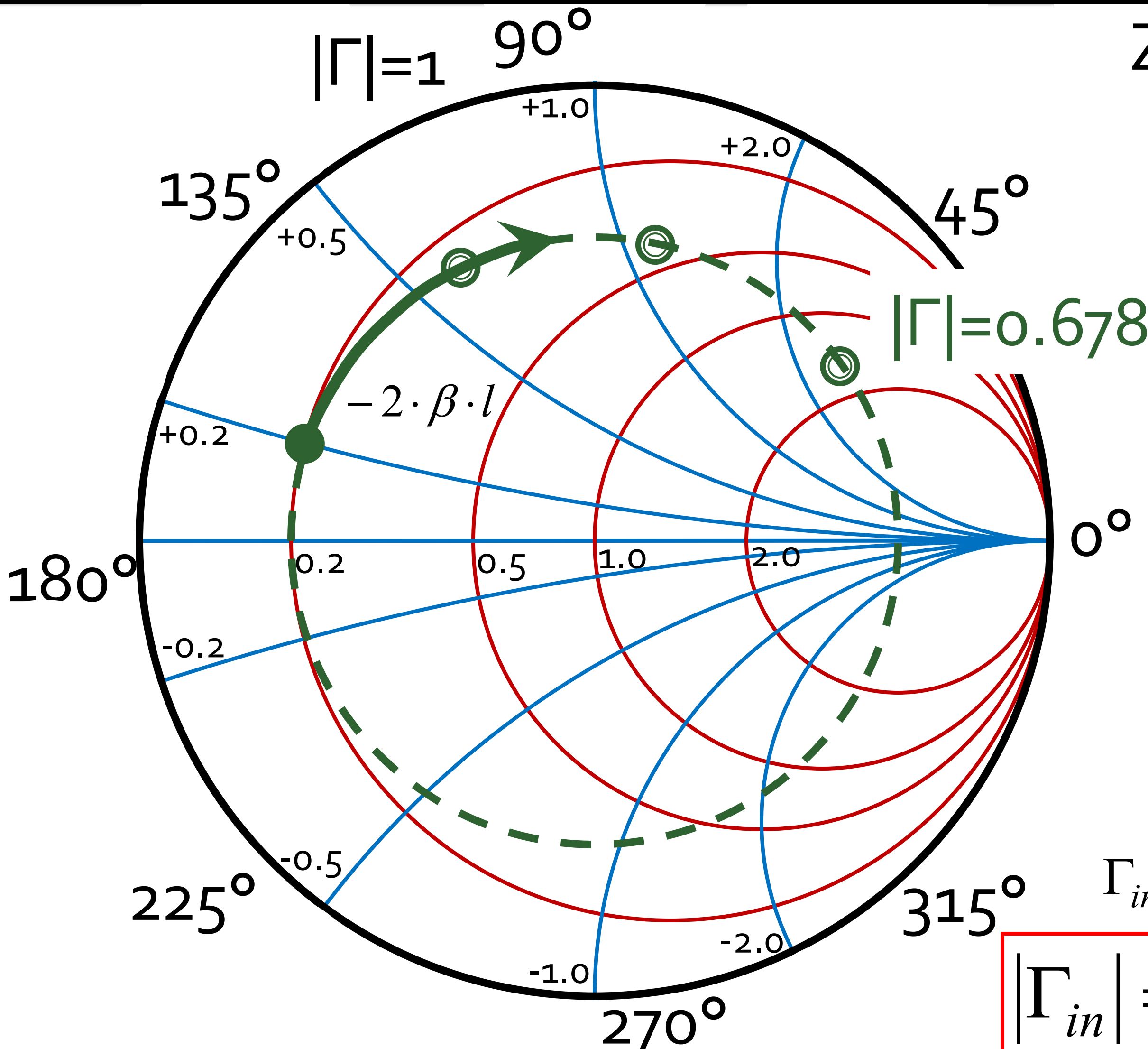
Impedance Matching

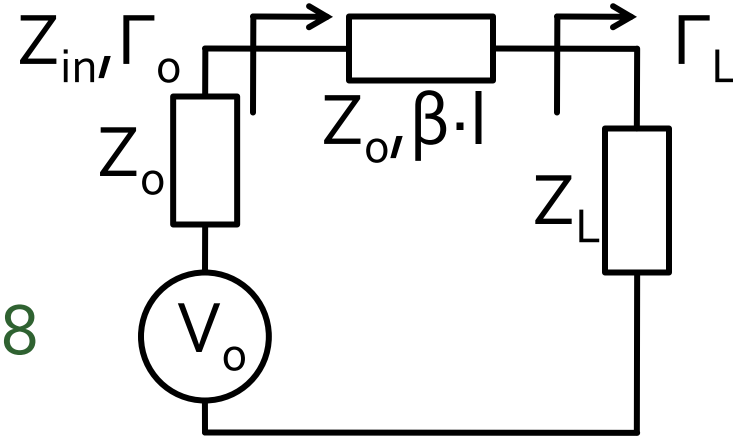
Impedance Matching with Stubs

Stub

- **Stub** (en)="rest, ciot, cotor, capăt" (ro)
- We avoid the necessity to use lumped elements
- Matching is achieved (with higher accuracy) using usual Zo transmission lines of the circuit
- We use one or more lengths of transmission line (stub) connected either in parallel or in series with the transmission feed line :
  - open-circuited
  - short-circuited
- Usually open-circuited transmission lines are easier to implement and are preferred

The Smith Chart, series transmission line, Z_o





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

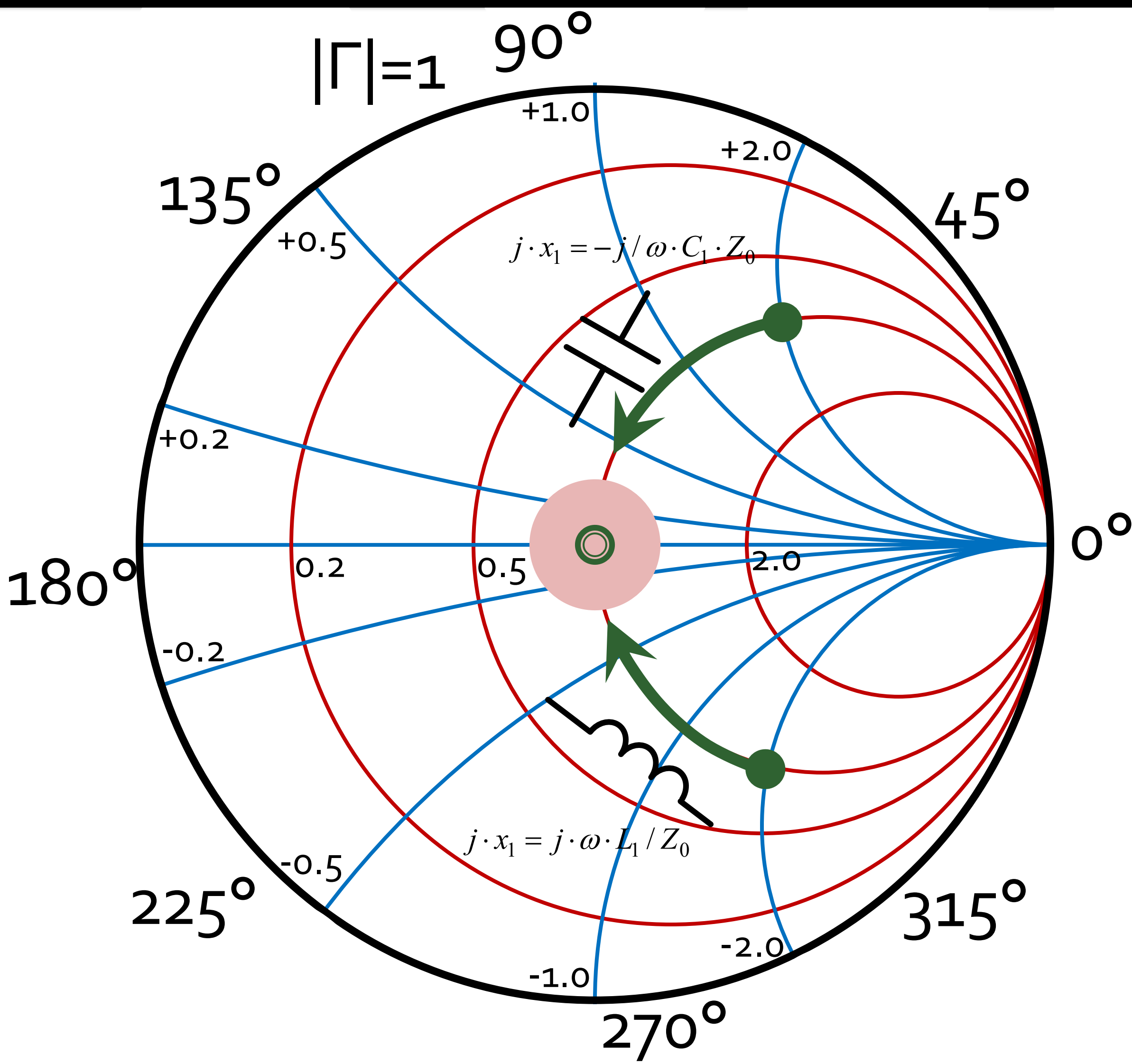
$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}$$

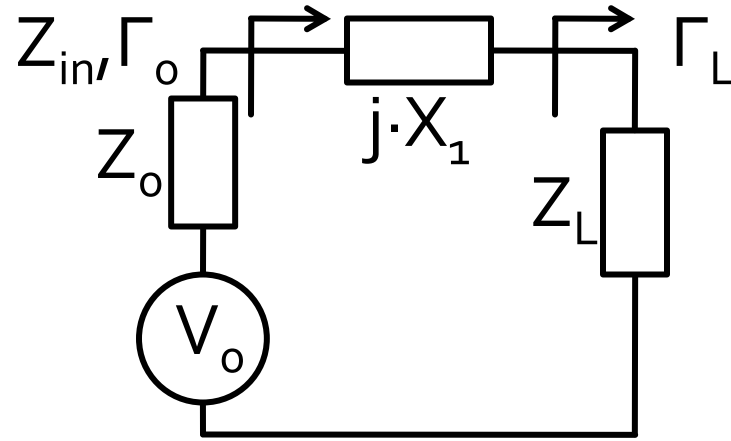
$$\Gamma_{in} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma_{in}| = |\Gamma_L|$$

$$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2\cdot\beta\cdot$$

Matching, series reactance





$$z_L = r_L + j \cdot x_L$$

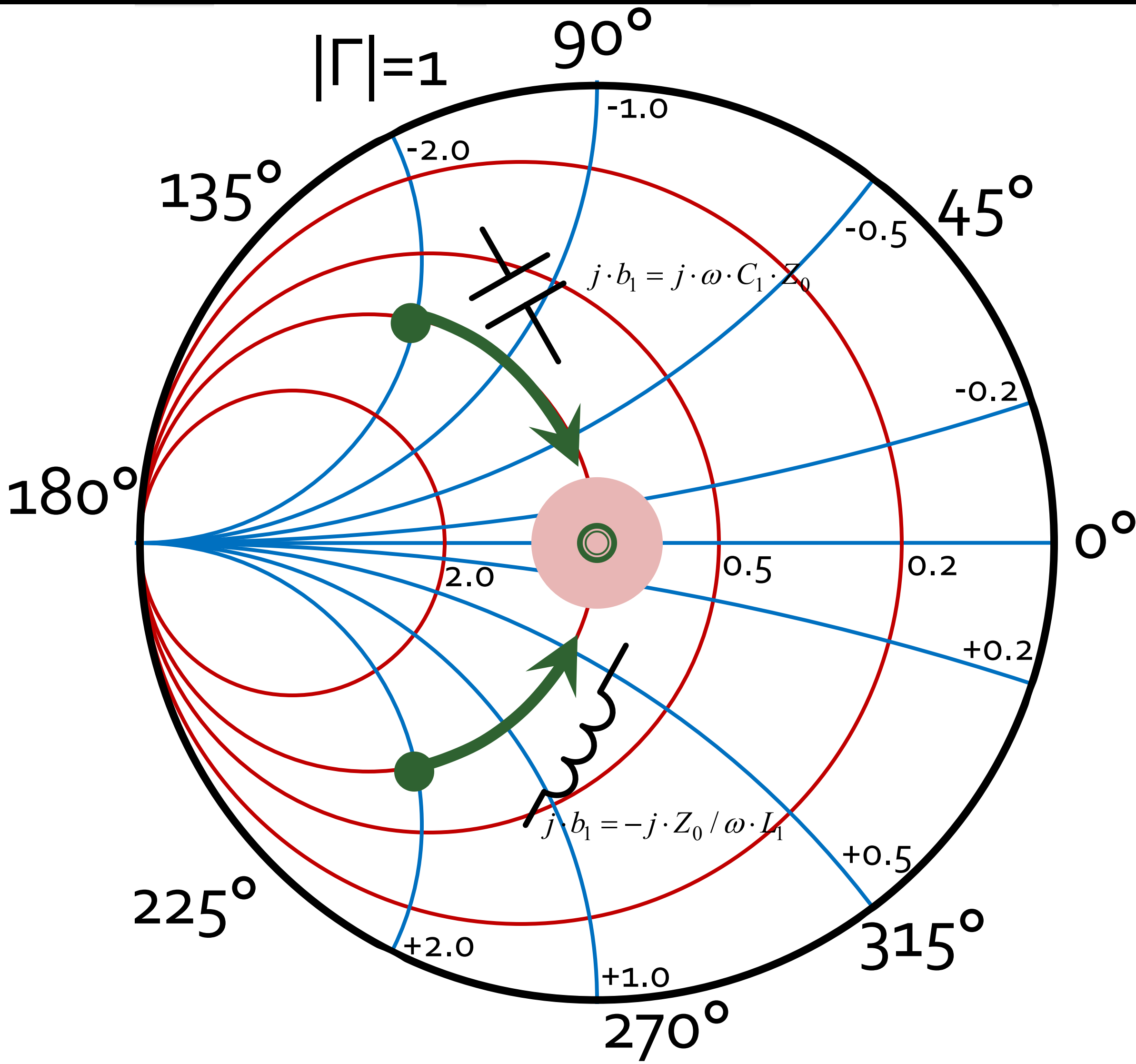
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

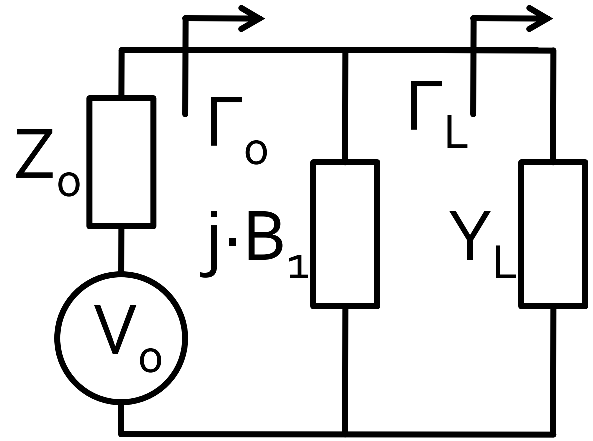
$$r_{in} = r_L$$

- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Matching, shunt susceptance





$$y_L = g_L + j\cdot b_L$$

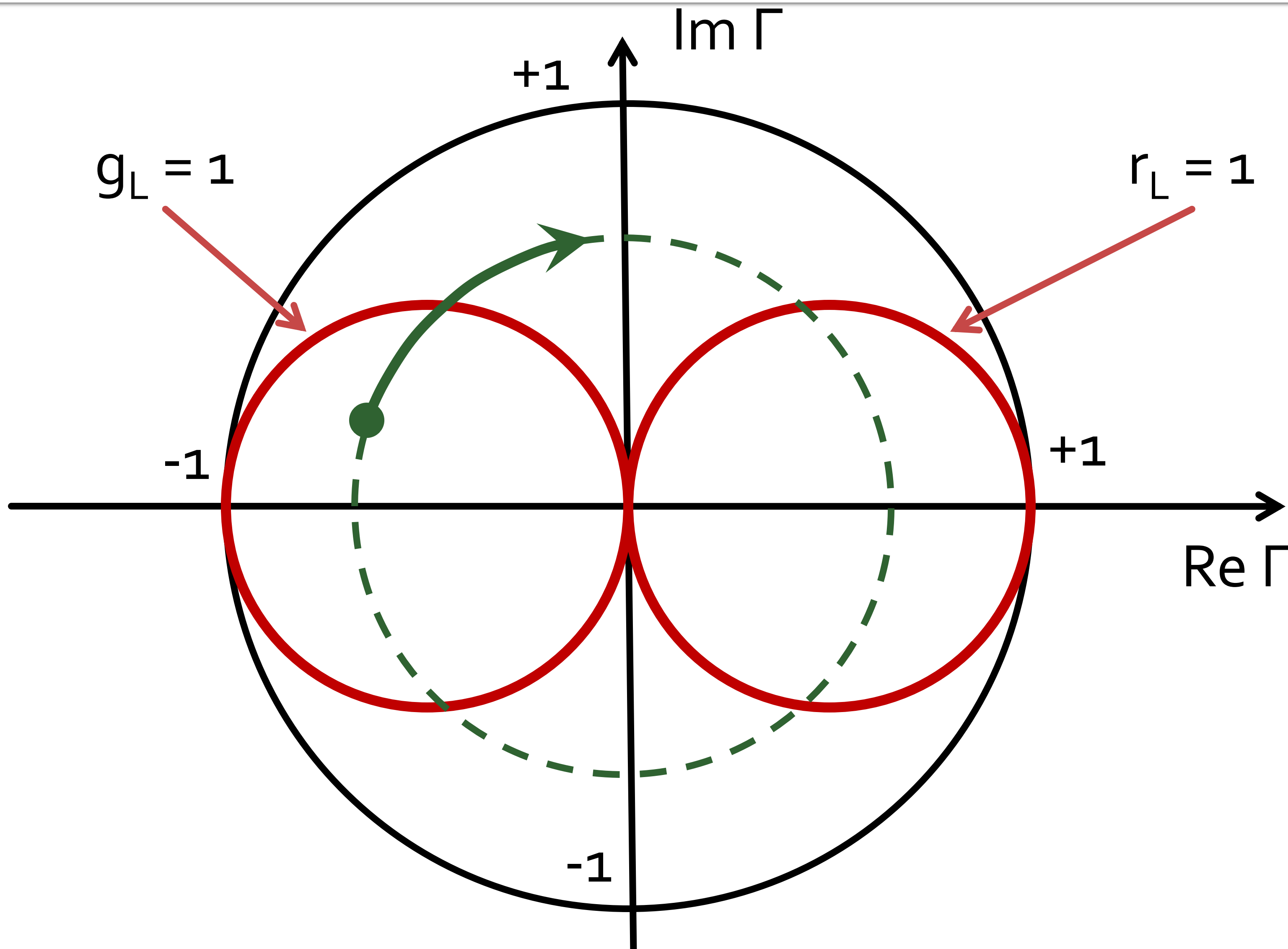
$$y_{in} = g_L + j\cdot(b_L + b_1)$$

$$g_{in} = g_L$$

- Match can be obtained **if and only if** $g_L = 1$
- we compensate the reactive part of the load

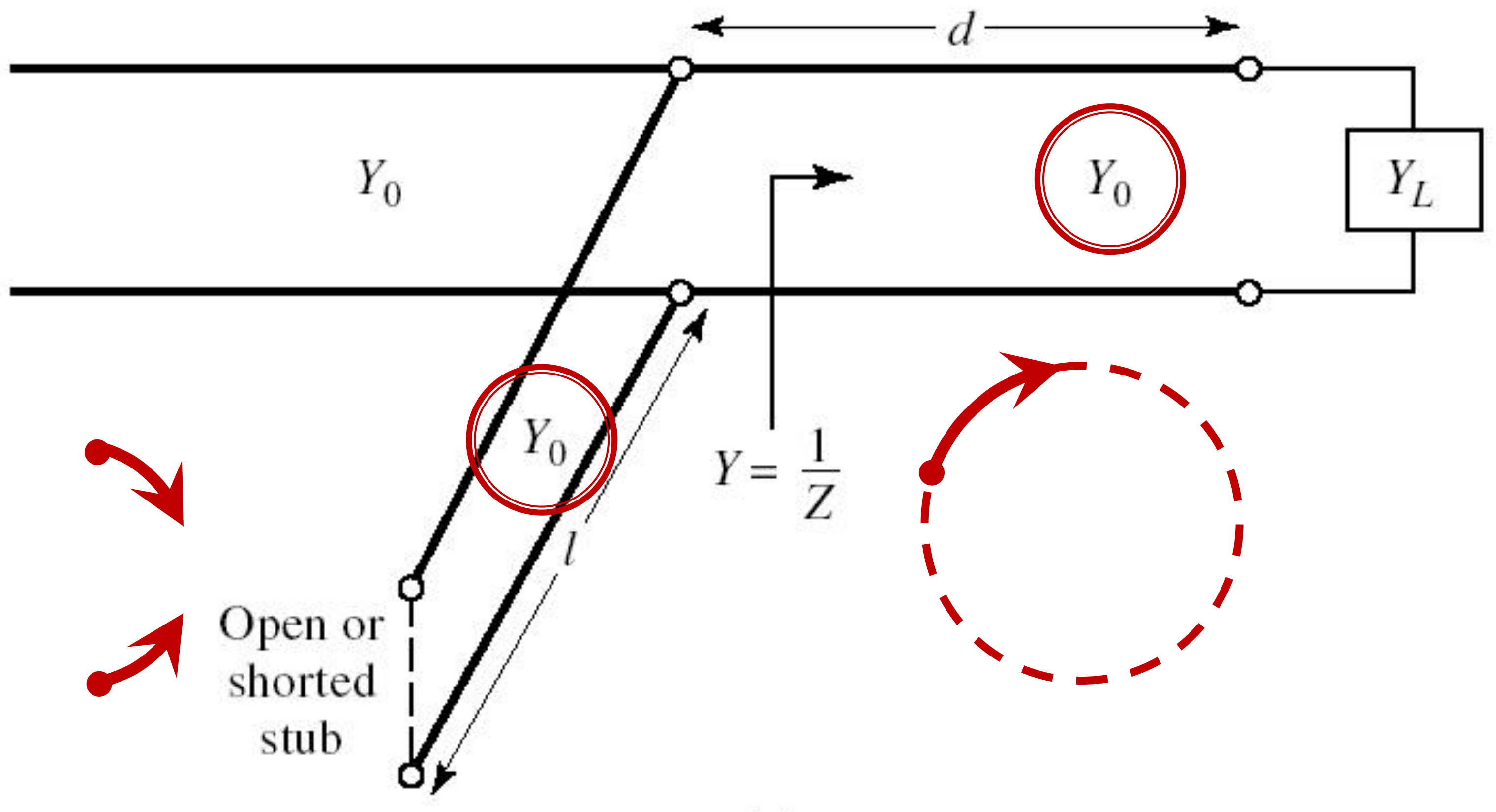
$$j\cdot b_1 = -j\cdot b_L$$

Smith chart, r=1 and g=1



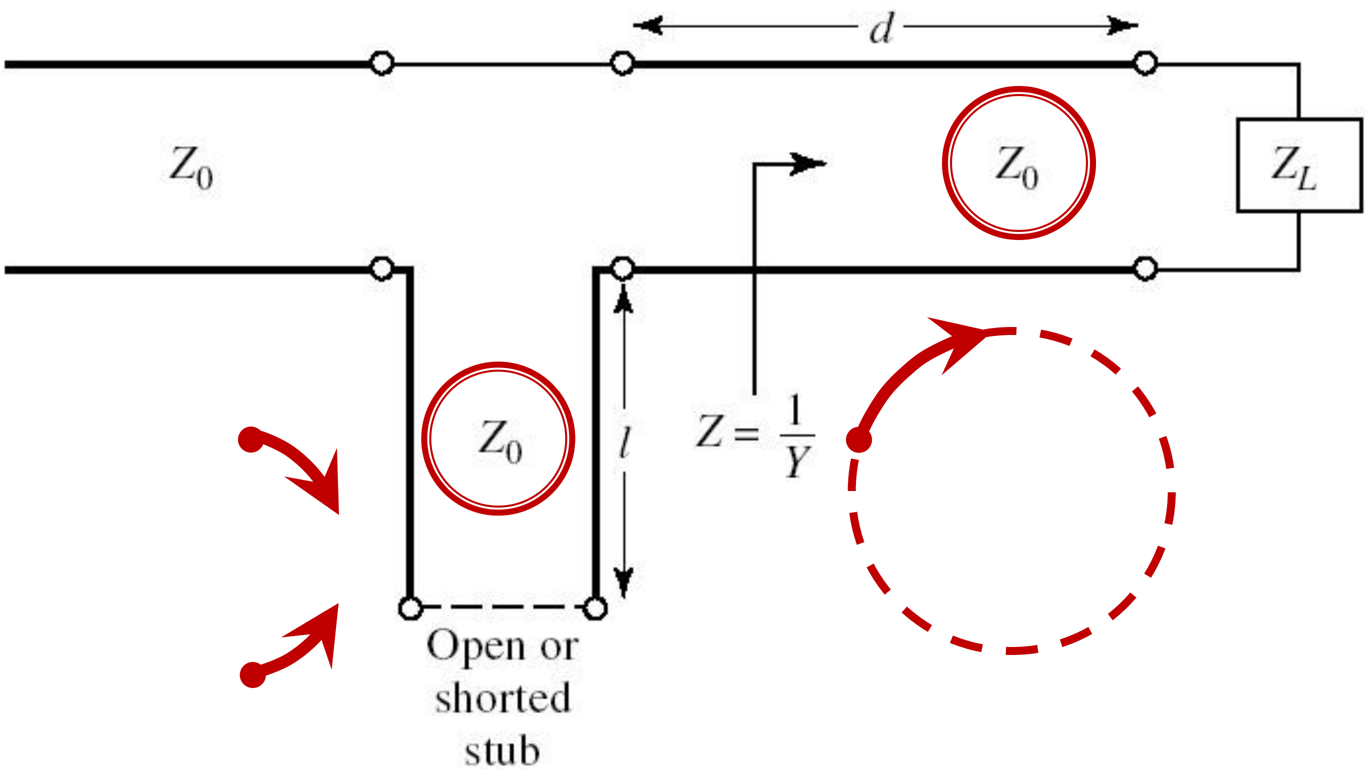
Single stub tuning

- Shunt Stub



Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

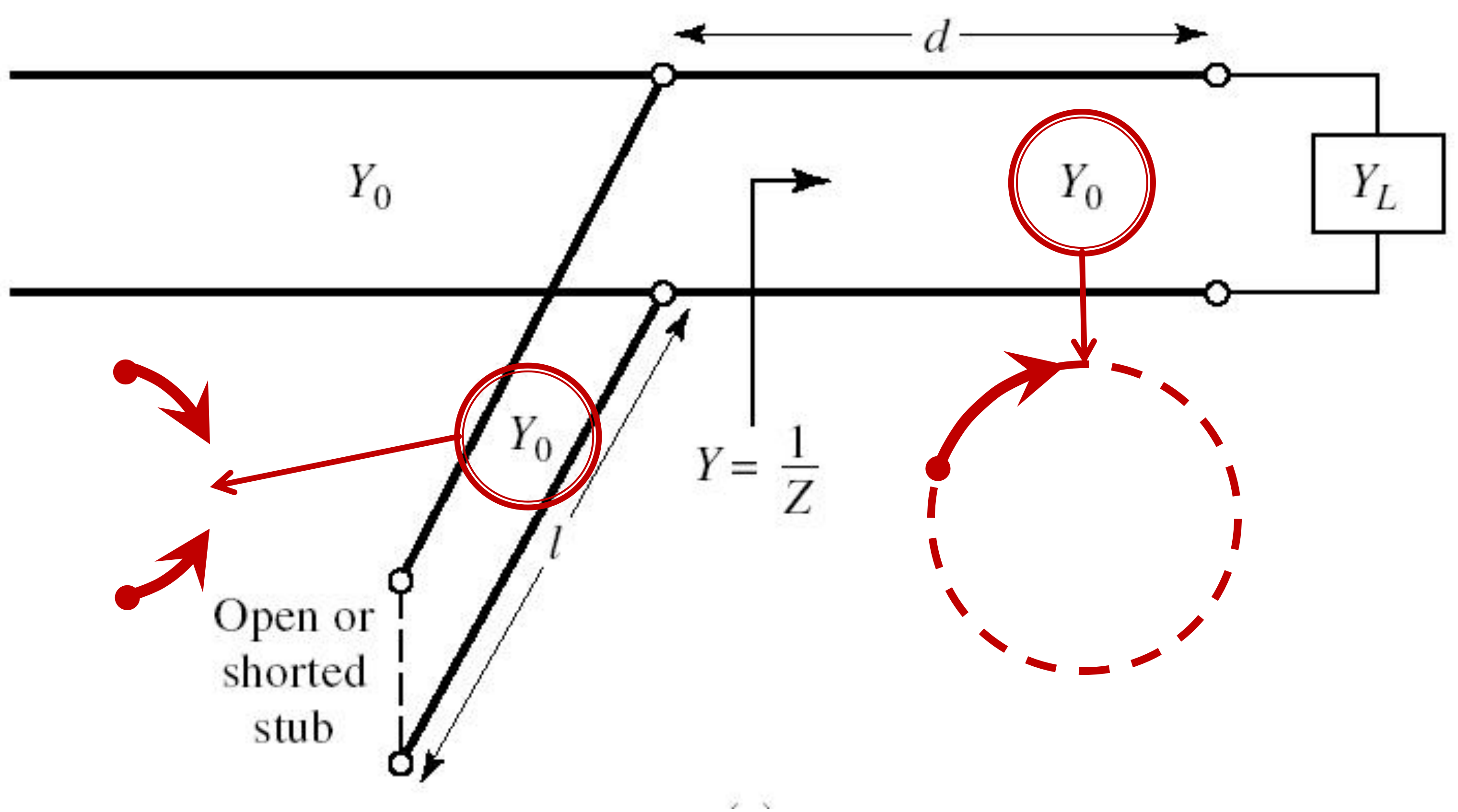


Shunt Stub

Sectiune de linie paralel

Case 1, Shunt Stub

- Shunt Stub



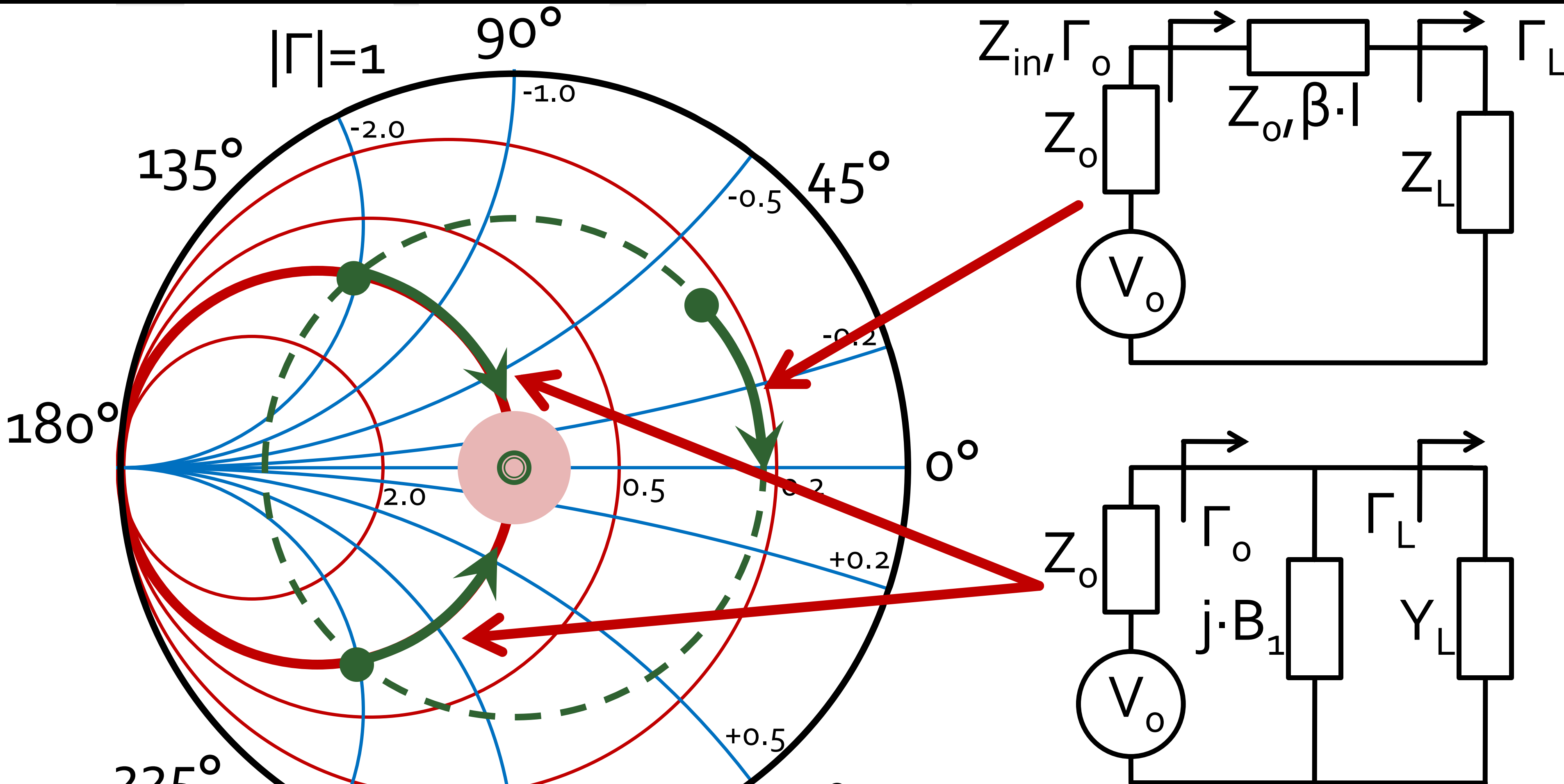
Case 1, Shunt Stub

- We use a series transmission line to move the reflection coefficient **on the circle** $g_L = 1$
- We compensate the remaining reactive part of the load with a shunt reactance to achieve match
- The shunt reactance is made with a stub which can be, as needed:
  - open-circuited
  - short-circuited

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \qquad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

Matching, series line + shunt susceptance

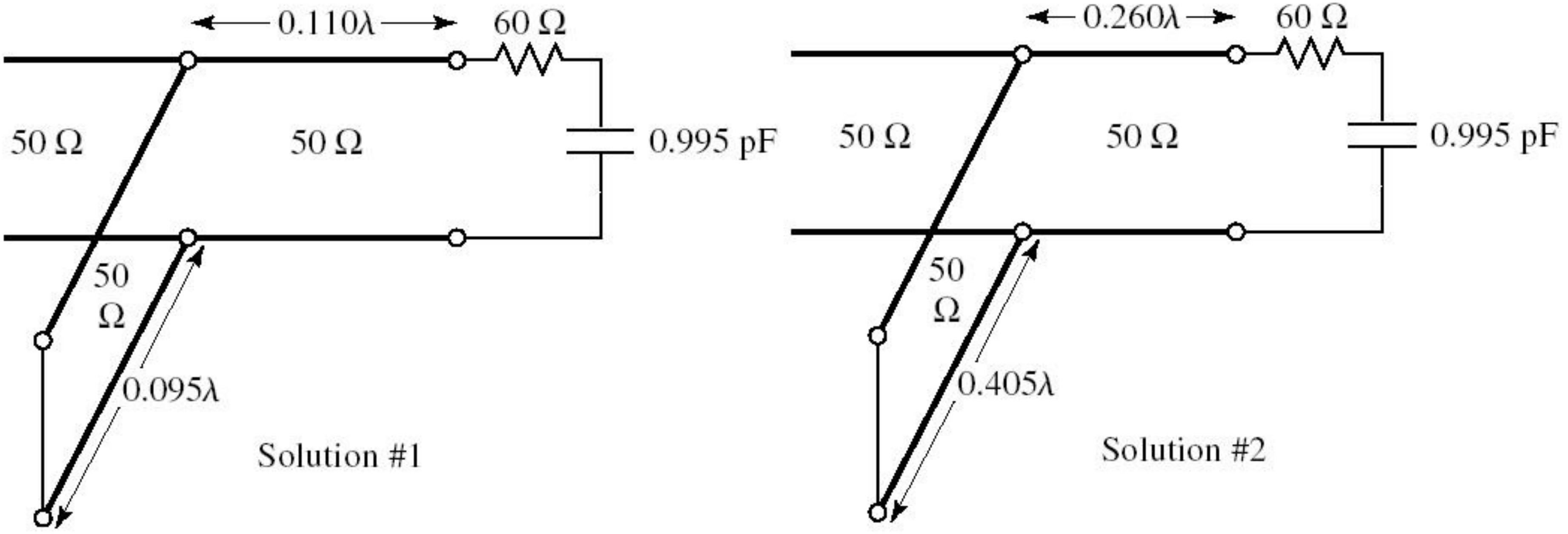


$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

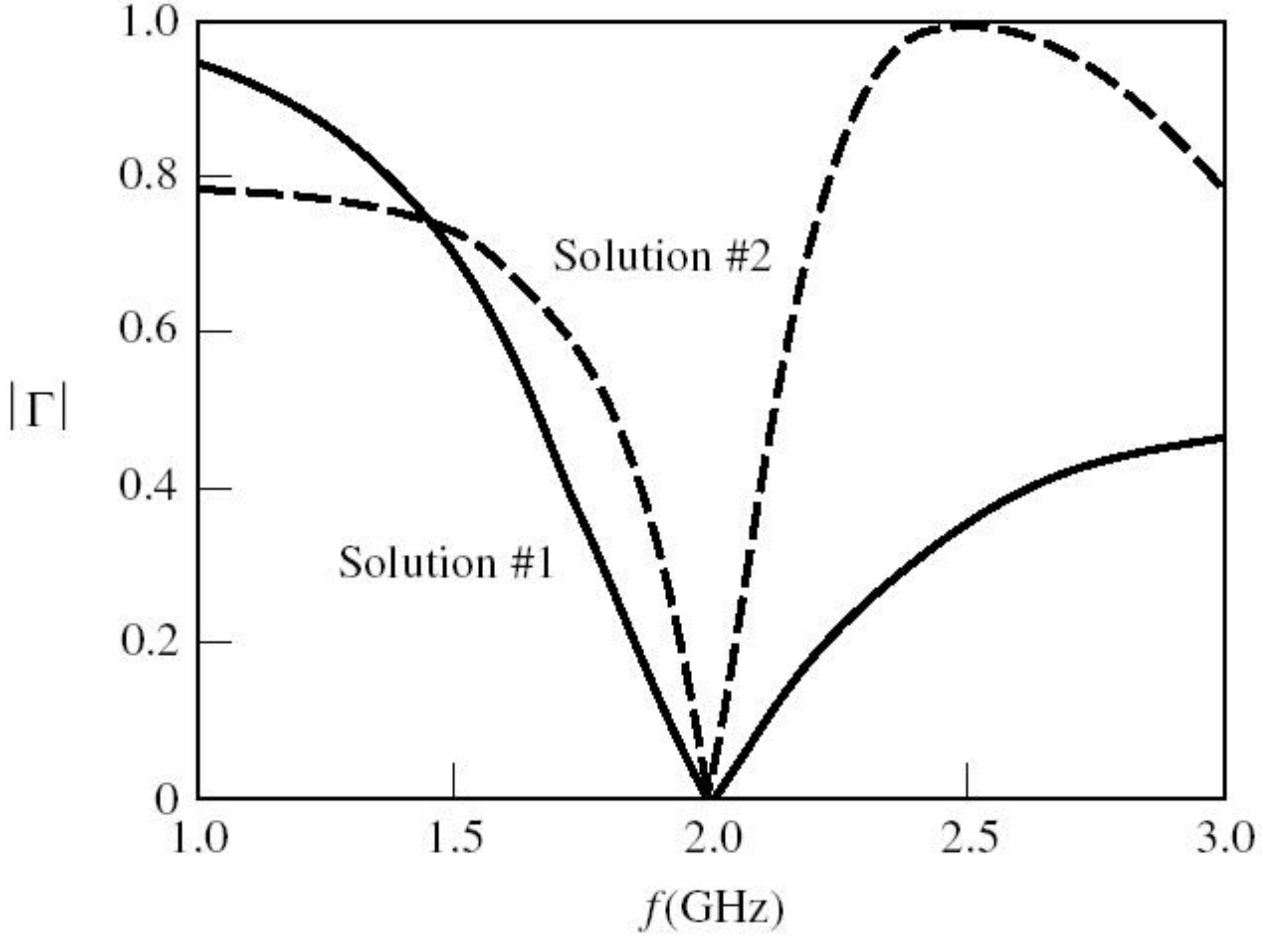
$$g_{in} = 1$$

Example, Shunt Stub, sc.

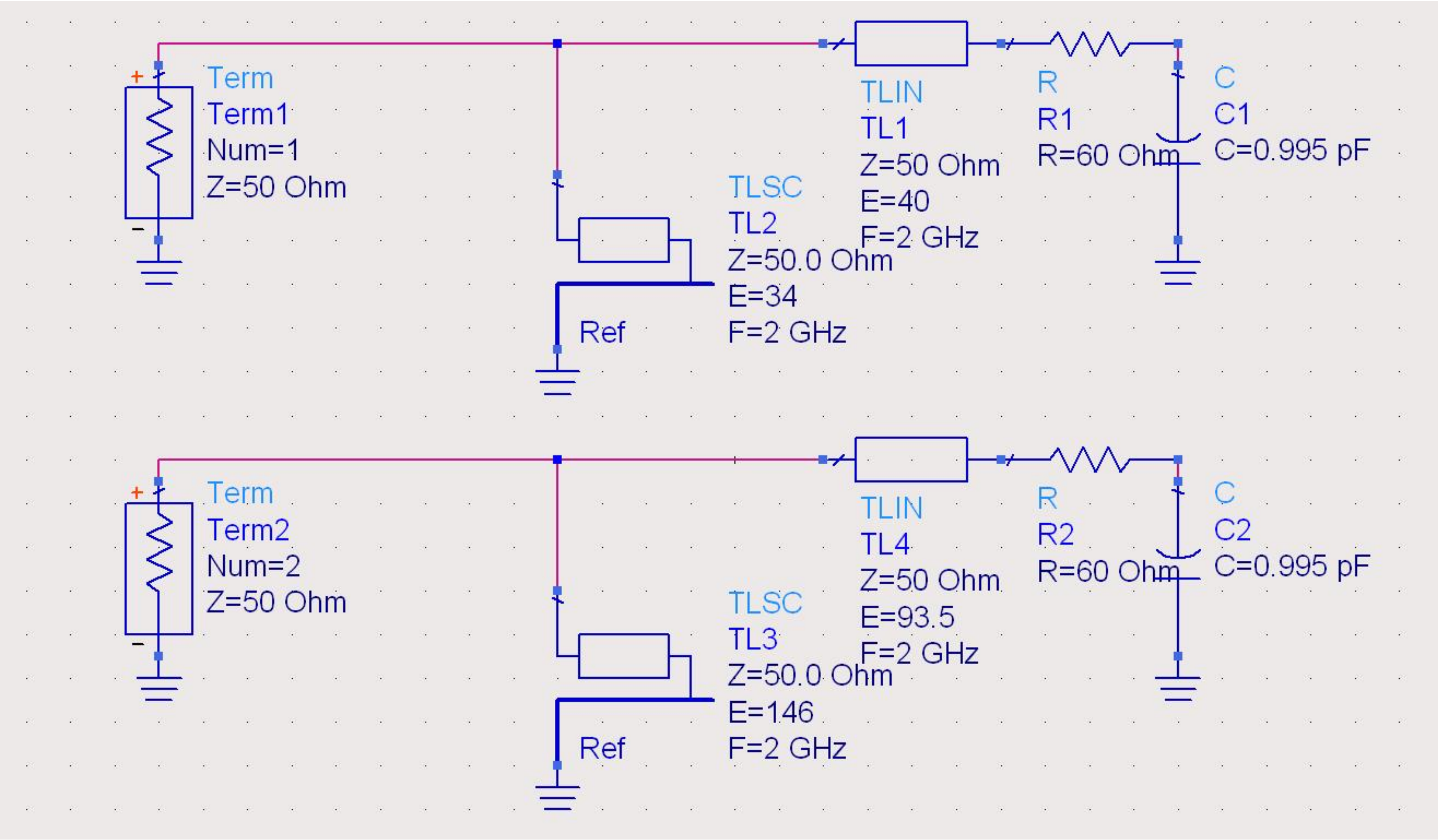
- load: 60 Ω series with 0.995 pF at 2GHz
- two possible solutions



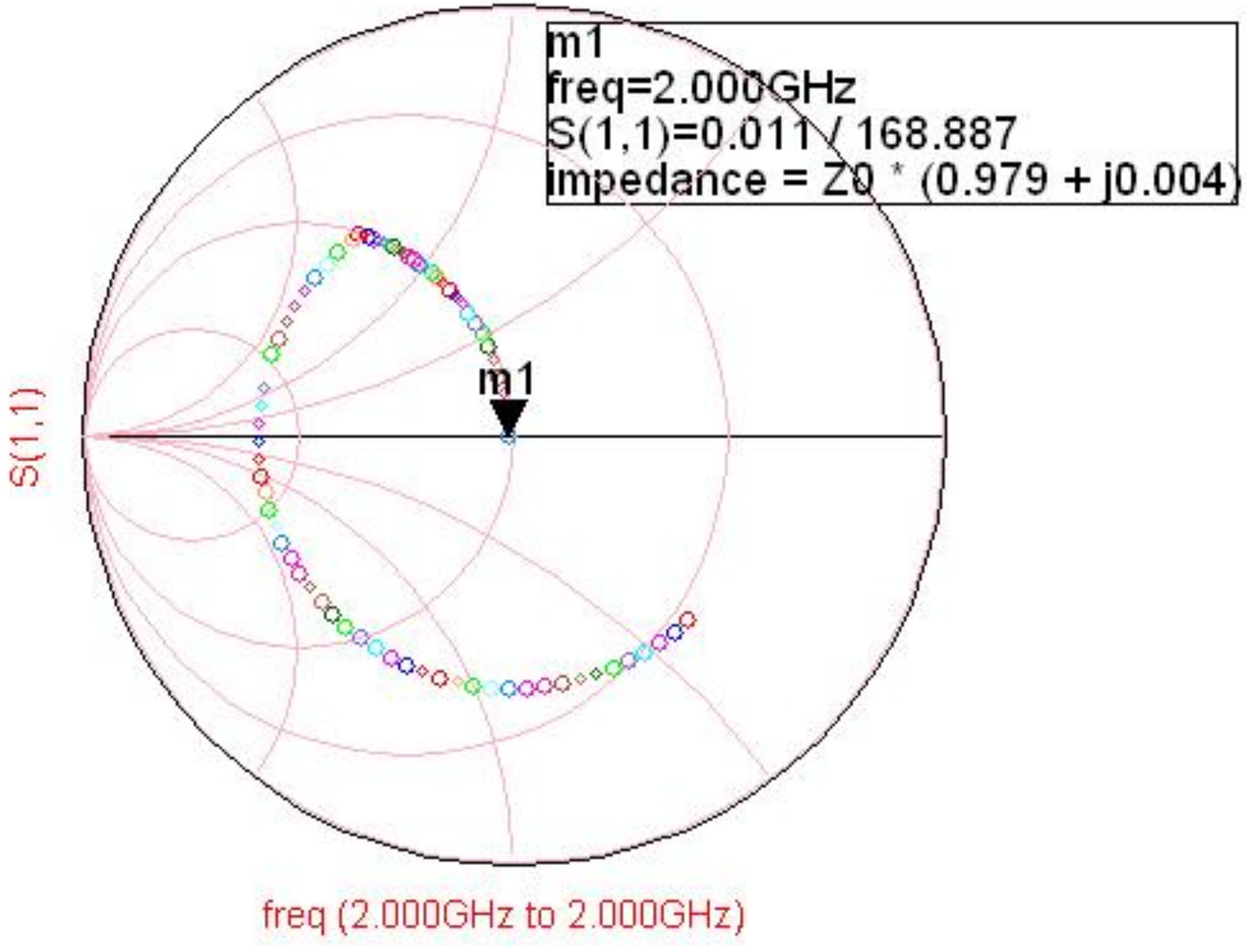
Example, Shunt Stub, sc.



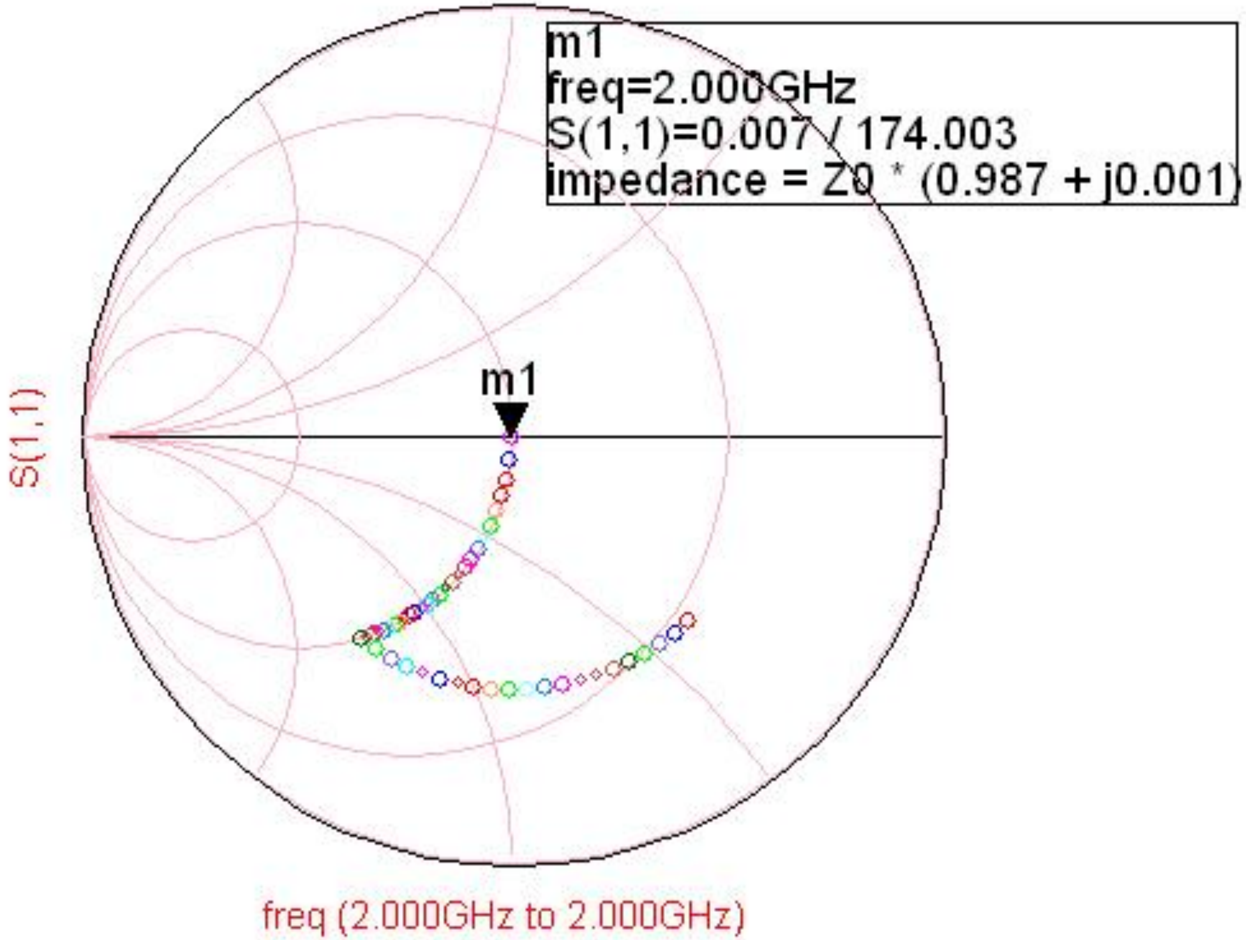
Example, Shunt Stub, sc.



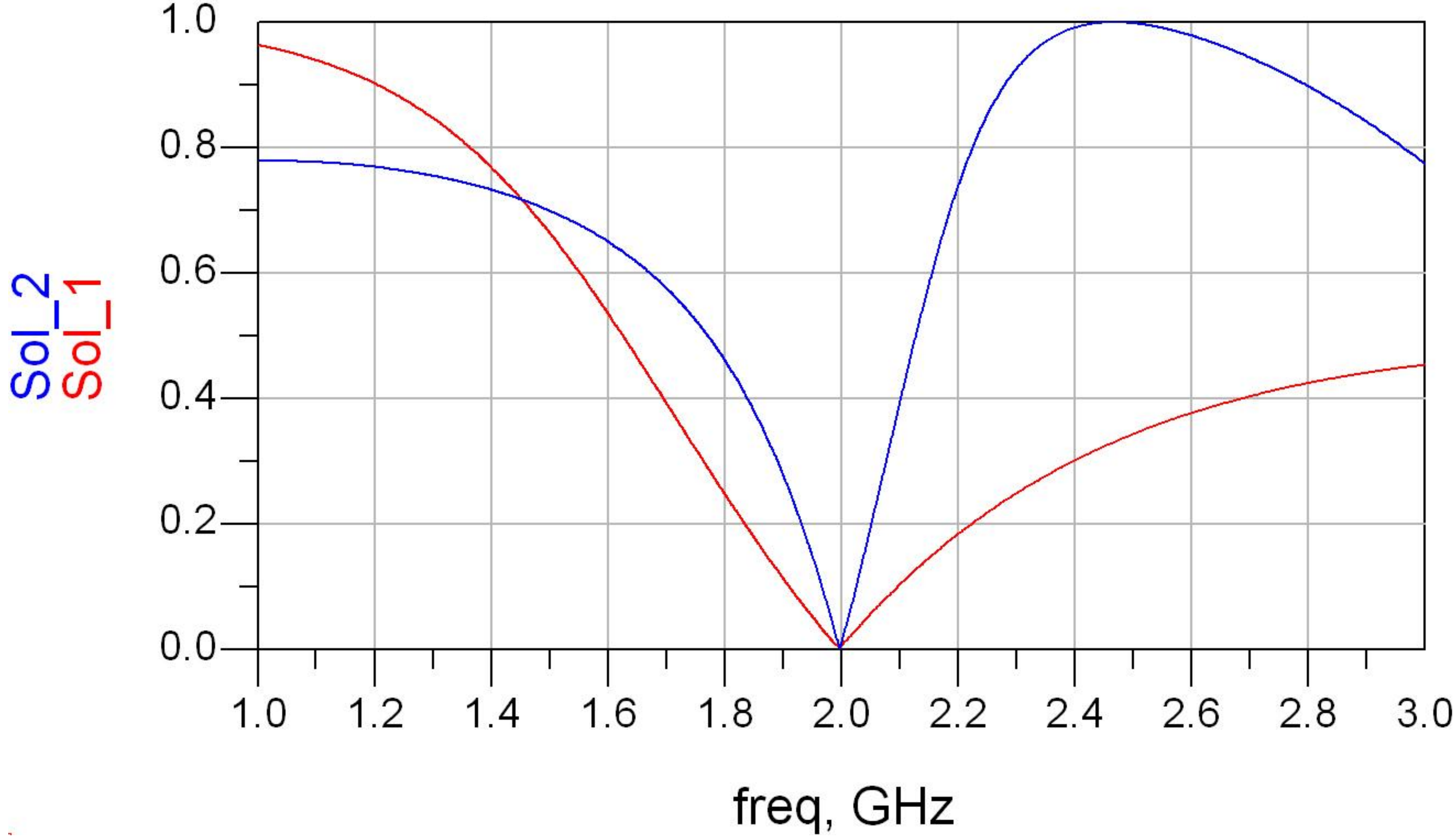
Example, Shunt Stub, sc.



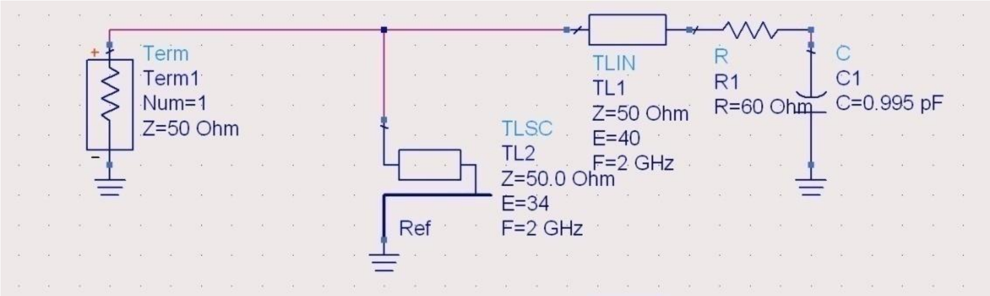
Example, Shunt Stub, sc.



Example, Shunt Stub, sc.



Example, Shunt Stub, sc.

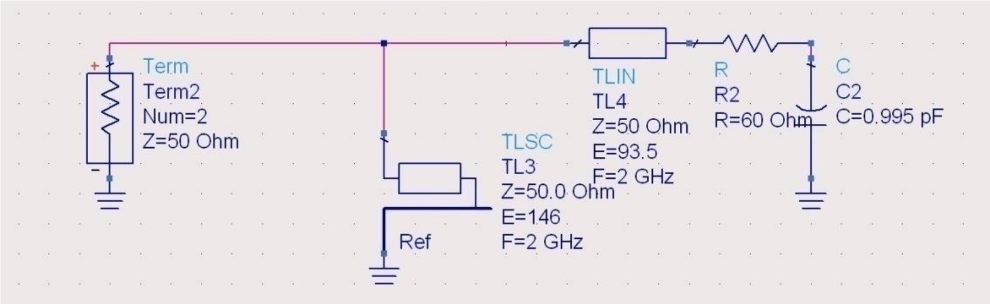


$$l_1 = \frac{40°}{360°} \cdot \lambda = 0.111 \cdot \lambda$$

$$l_2 = \frac{34°}{360°} \cdot \lambda = 0.094 \cdot \lambda$$

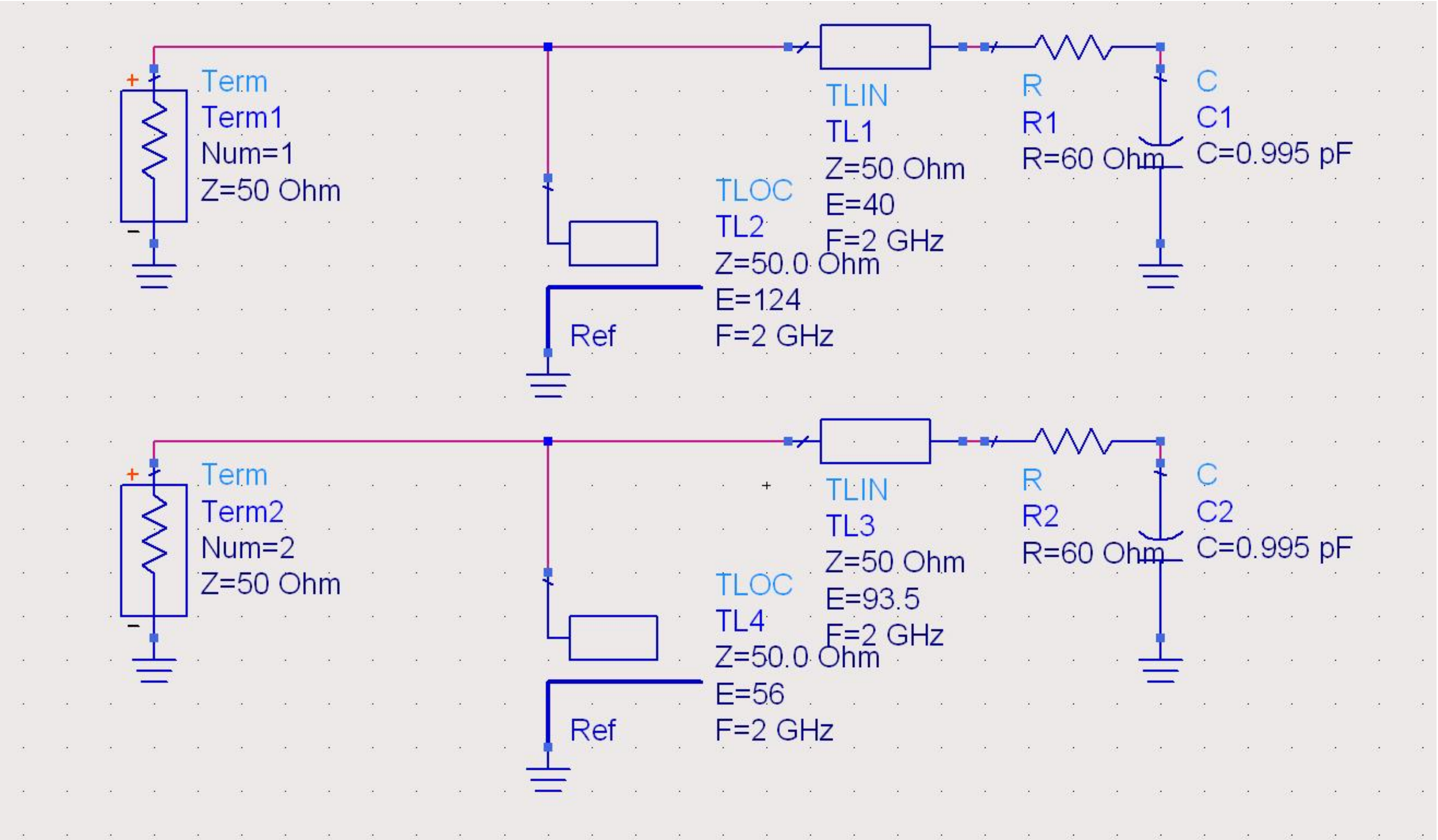
$$l_1 = \frac{93.5°}{360°} \cdot \lambda = 0.260 \cdot \lambda$$

$$l_2 = \frac{146°}{360°} \cdot \lambda = 0.406 \cdot \lambda$$

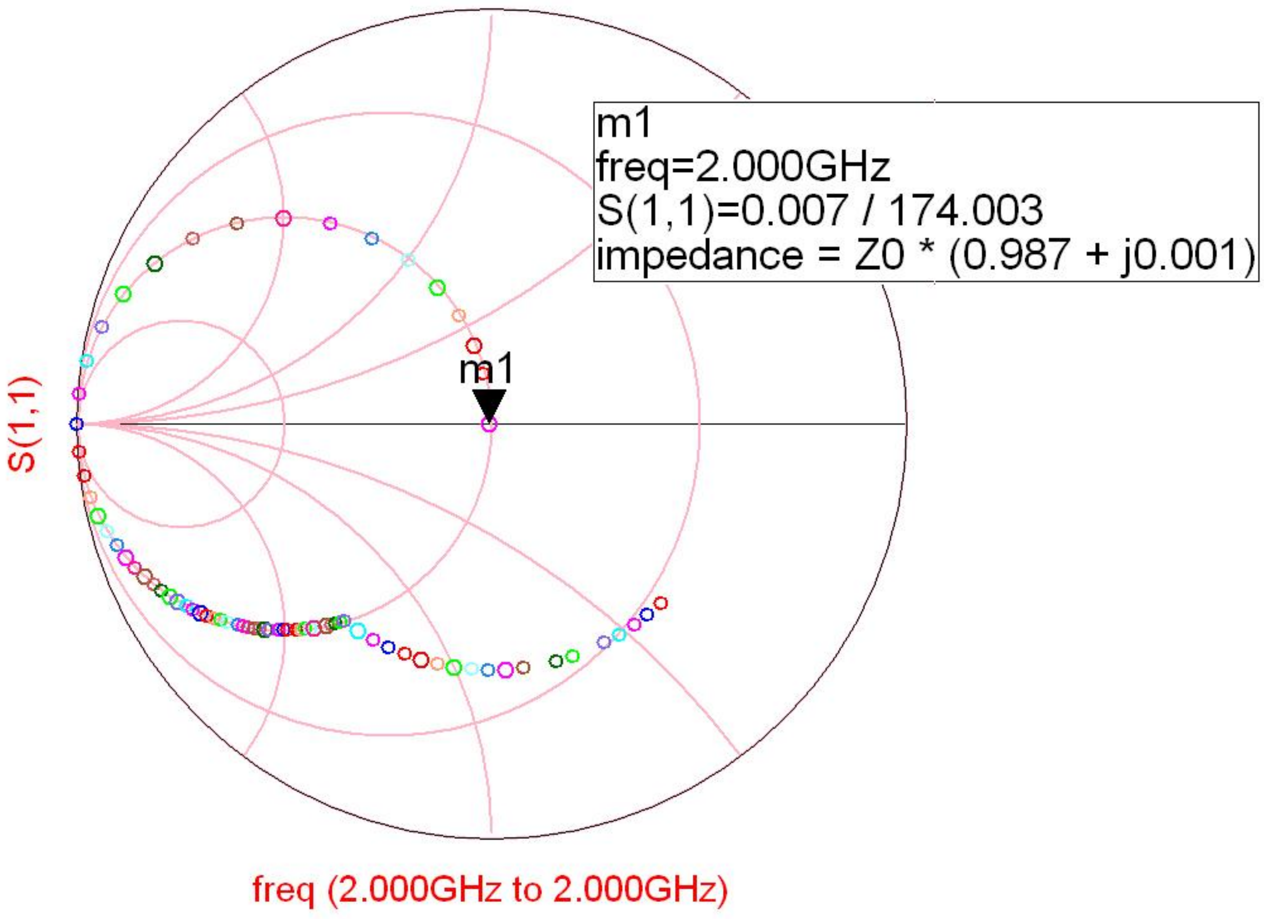


Example, Shunt Stub, oc.

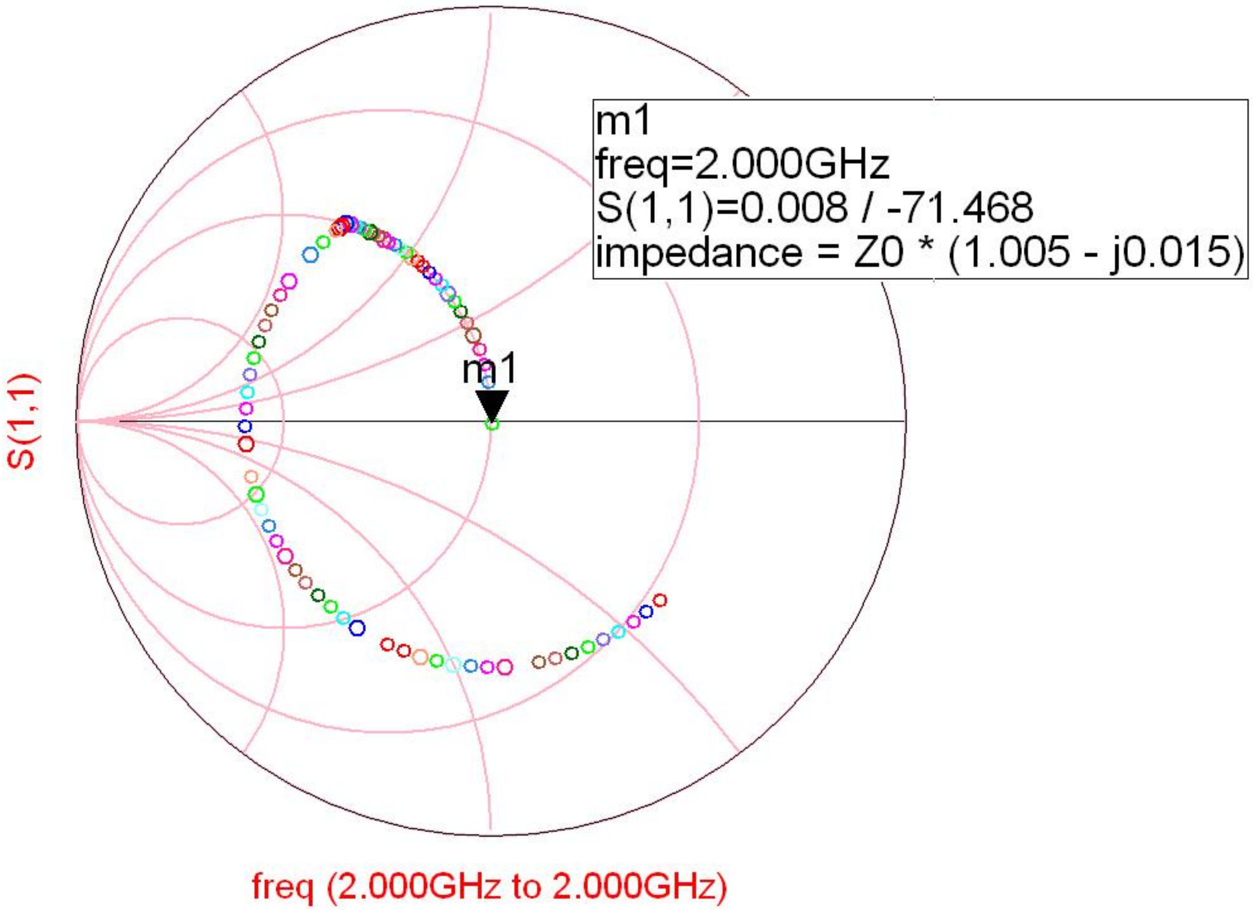
- load: 60 Ω series with 0.995 pF at 2GHz
- two possible solutions



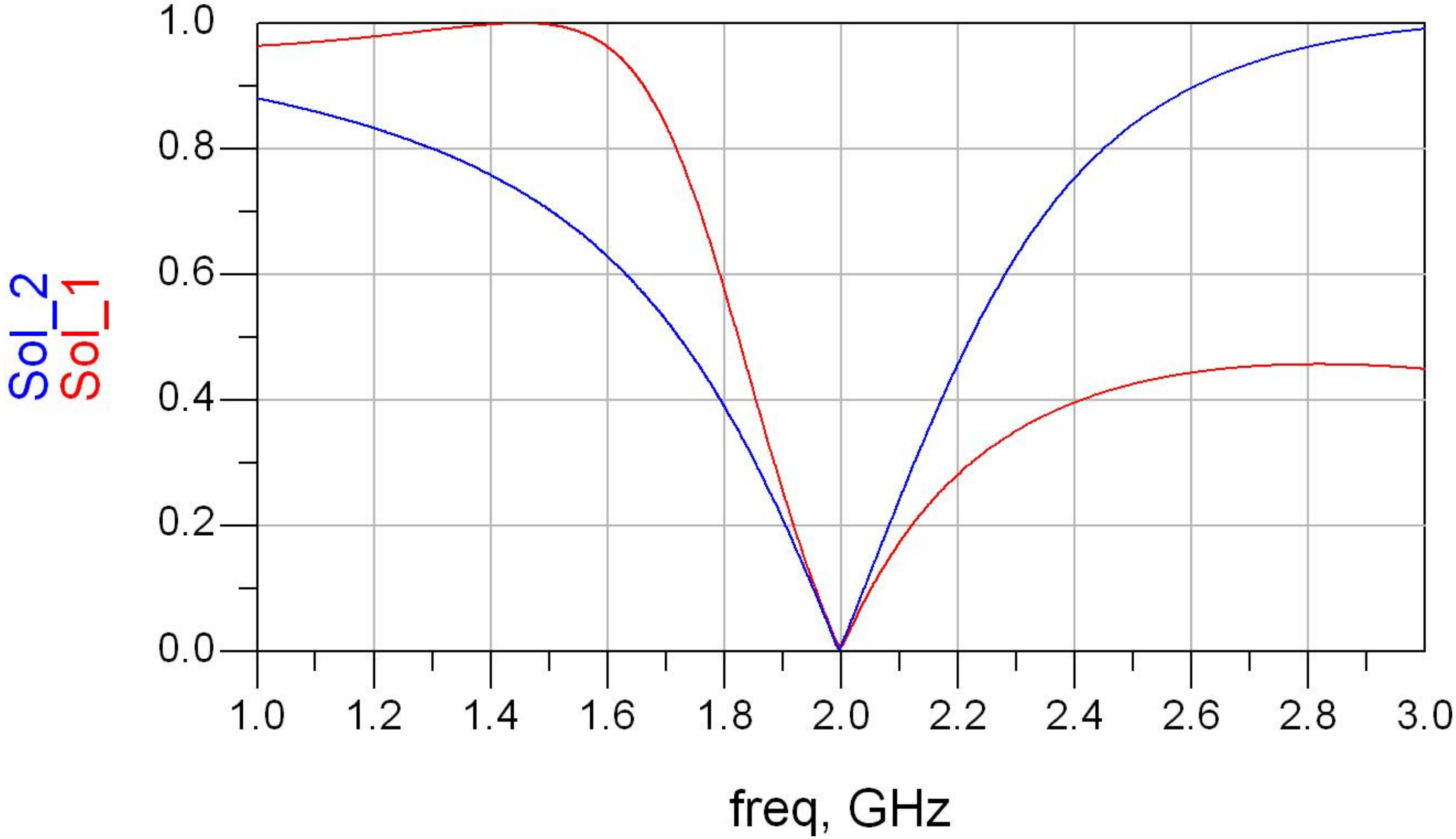
Example, Shunt Stub, oc.



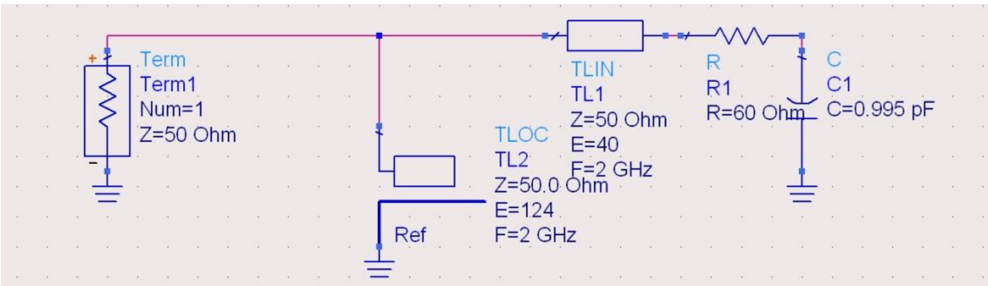
Example, Shunt Stub, oc.



Example, Shunt Stub, oc.

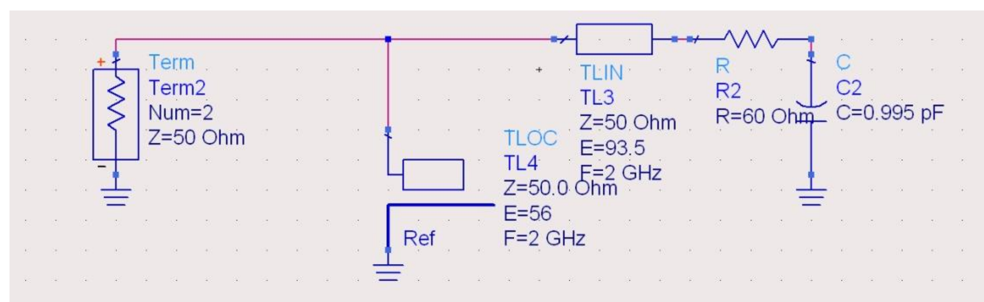


Example, Shunt Stub, oc.



$$l_1 = \frac{40^\circ}{360^\circ} \cdot \lambda = 0.111 \cdot \lambda$$

$$l_2 = \frac{124^\circ}{360^\circ} \cdot \lambda = 0.344 \cdot \lambda = 0.094 \cdot \lambda + \frac{\lambda}{4}$$



$$l_1 = \frac{93.5^\circ}{360^\circ} \cdot \lambda = 0.260 \cdot \lambda$$

$$l_2 = \frac{56^\circ}{360^\circ} \cdot \lambda = 0.156 \cdot \lambda = 0.406 \cdot \lambda - \frac{\lambda}{4}$$

Shunt Stub, some notes

- mathematical functions which offer the input impedance in a stub are periodic functions of l, tan/cot based functions

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \qquad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

  - adding or subtracting

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  doesn't change the result (full rotation around the Smith Chart – hence the 0.5λ gradation of the circumference of the diagram)

Shunt Stub, some notes

- adding or subtracting λ/4 transforms the impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \qquad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

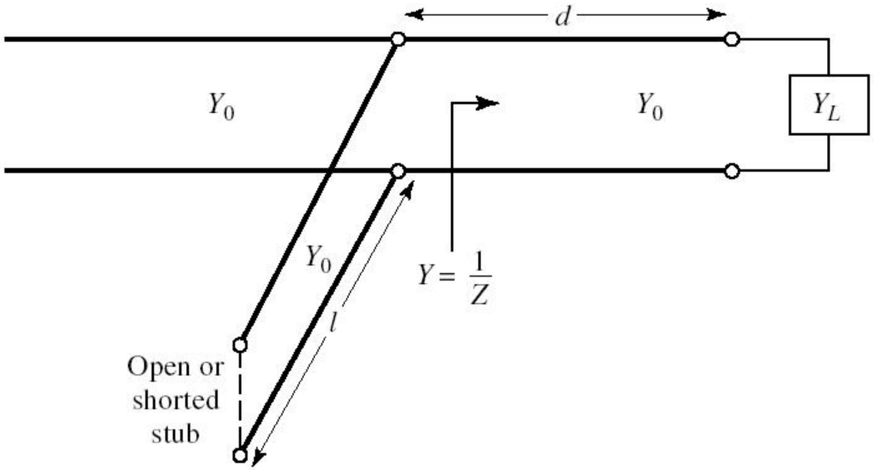
$$\tan \beta \cdot \left(l + \frac{\lambda}{4} \right) = \tan\left(\beta \cdot l + \frac{\pi}{2} \right) = \frac{\sin(\beta \cdot l + \pi/2)}{\cos(\beta \cdot l + \pi/2)} = \frac{\cos \beta \cdot l}{-\sin \beta \cdot l} = -\cot \beta \cdot l$$

  from the open-circuited value to the short-circuited one and vice versa
- For tuning in ADS it's better to start from the neutral point (value of the electrical length of the line which doesn't influence the circuit)
  - series line: $E = \beta \cdot l = 0$
  - shunt stub: $Z_{in} \rightarrow \infty, \quad \tan \beta \cdot l / \cot \beta \cdot l \rightarrow \infty, \quad E = 90° / 0°$

Analytical solution

Shunt Stub

Analytical solution, impedances



$$Z_L = \frac{1}{Y_L} = R_L + j \cdot X_L$$

$$Z = Z_0 \cdot \frac{(R_L + j \cdot X_L) + j \cdot Z_0 \cdot t}{Z_0 + j \cdot (R_L + j \cdot X_L) \cdot t}$$

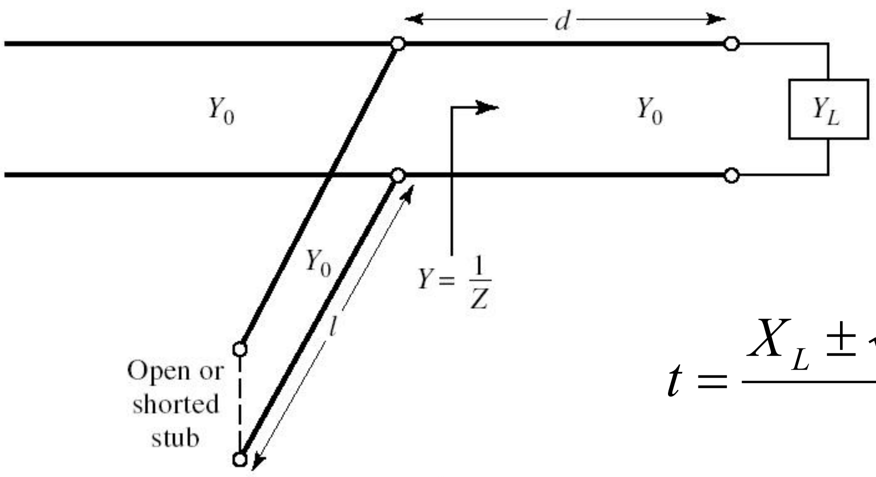
$$t \overset{not}{=} \tan \beta \cdot d \qquad Y = G + j \cdot B = \frac{1}{Z}$$

$$G = \frac{R_L \cdot (1 + t^2)}{R_L^2 + (X_L + Z_0 \cdot t)^2} \qquad B = \frac{R_L^2 \cdot t - (Z_0 - X_L \cdot t) \cdot (X_L + Z_0 \cdot t)}{Z_0 \cdot \left[R_L^2 + (X_L + Z_0 \cdot t)^2\right]}$$

- *d* (which implies t) is chosen so that: $G = Y_0 = \frac{1}{Z_0}$

$$Z_0 \cdot (R_L - Z_0) \cdot t^2 - 2 \cdot X_L \cdot Z_0 \cdot t + \left(R_L \cdot Z_0 - R_L^2 - X_L^2\right) = 0$$

Analytical solution



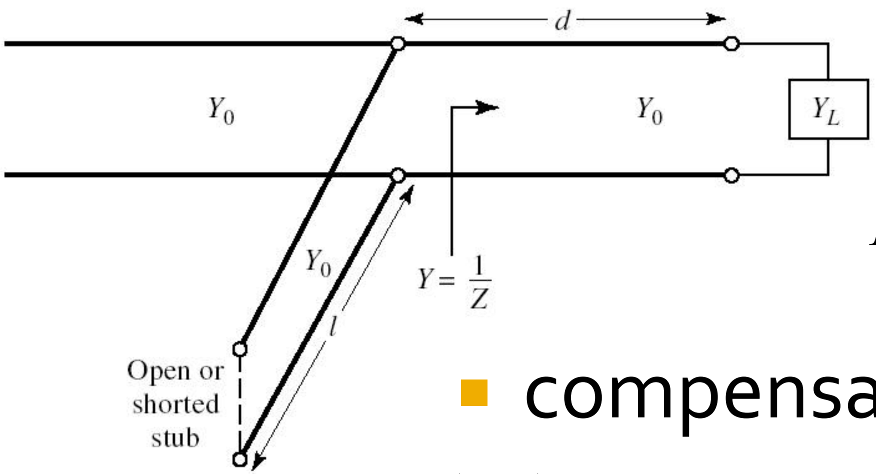
$$t = -\frac{X_L}{2 \cdot Z_0}, \quad R_L = Z_0$$

$$t = \frac{X_L \pm \sqrt{R_L \cdot \left[(Z_0 - R_L)^2 + X_L^2 \right] / Z_0}}{R_L - Z_0} \quad R_L \neq Z_0$$

- second grade equation, 2 solutions possible
- *d* computed from *t*

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi} \cdot \arctan t & t \geq 0 \\ \frac{1}{2\pi} \cdot (\pi + \arctan t) & t < 0 \end{cases}$$

Analytical solution



$$B_S = -B$$

$$B = \frac{R_L^2 \cdot t - (Z_0 - X_L \cdot t) \cdot (X_L + Z_0 \cdot t)}{Z_0 \cdot \left[R_L^2 + (X_L + Z_0 \cdot t)^2\right]}$$

- compensating susceptance is:

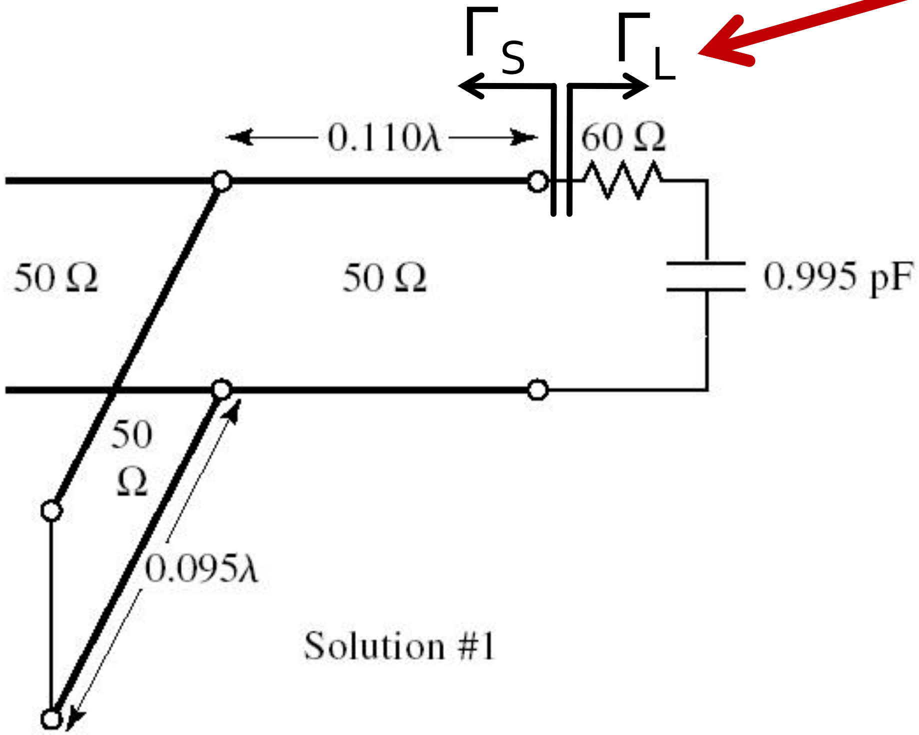
$$\frac{l_{oc}}{\lambda} = \frac{1}{2\pi} \cdot \arctan\left(\frac{B_S}{Y_0}\right) = \frac{-1}{2\pi} \cdot \arctan\left(\frac{B}{Y_0}\right)$$

$$\frac{l_{sc}}{\lambda} = \frac{-1}{2\pi} \cdot \arctan\left(\frac{Y_0}{B_S}\right) = \frac{1}{2\pi} \cdot \arctan\left(\frac{Y_0}{B}\right)$$

- for **negative lengths** we add λ/2

Analytical solution, reflection coefficient

- load: 60 Ω series with 0.995 pF at 2GHz



$$Z_L = R_L + \frac{1}{j\cdot\omega\cdot C_L} = 60\Omega - j\cdot 79.977\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.405 - j\cdot 0.432$$

$$Y_L = \frac{1}{Z_L} = 0.006S + j\cdot 0.008S$$

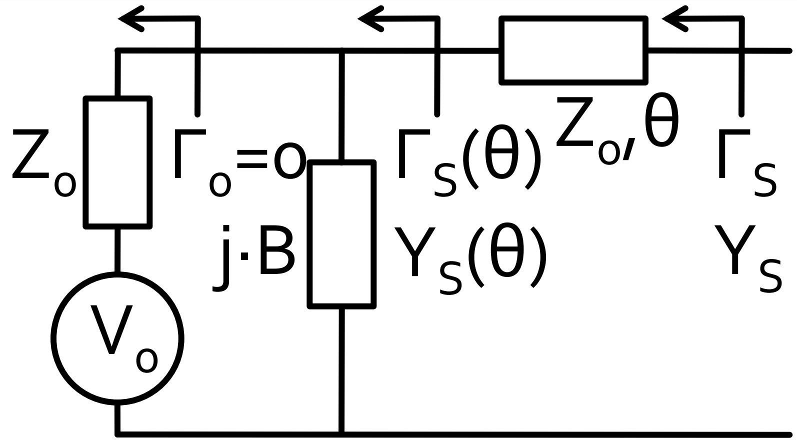
$$y_L = \frac{Y_L}{Y_0} = 0.3 + j\cdot 0.4$$

- matching requires obtaining conjugate value for Γ

$$\Gamma_S = \Gamma_L^* = 0.405 + j\cdot 0.432 \qquad \Gamma_S = 0.593\angle 46.85^\circ \qquad |\Gamma_S| = 0.593;\quad \varphi = 46.85^\circ$$

Analytical solution, Γ

- series line
  - electrical length E= β·l=θ
  - moves the reflection coefficient on the circle g=1
- shunt stub
  - electrical length E= $\beta \cdot l_{sp} = \theta_{sp}$
  - moves the reflection coefficient to the center of the Smith Chart (Γ_o=0)



$$y_S = \frac{Y_S}{Y_0} = Y_S \cdot Z_0 = Y_S \cdot 50\Omega$$

$$y_S = \frac{1-\Gamma_S}{1+\Gamma_S} = 0.3 - j \cdot 0.4$$

$$\Gamma_S(\theta) = [\Gamma_L(\theta)]^* = \left[\Gamma_L \cdot e^{-2j\theta}\right]^*$$

$$\Gamma_S(\theta) = \Gamma_L^* \cdot e^{2j\theta} = \Gamma_S \cdot e^{2j\theta}$$

$$y_S(\theta) = \frac{1-\Gamma_S(\theta)}{1+\Gamma_S(\theta)} = \frac{1-\Gamma_S \cdot e^{2j\theta}}{1+\Gamma_S \cdot e^{2j\theta}}$$

A.S., Γ, series line, proof

- After the series line with electrical length θ:

$$\mathrm{Re}[y_S(\theta)] = 1 \qquad \mathrm{Im}[y_S(\theta)] = B$$

$$\mathrm{Re}[y_S(\theta)] = \frac{1}{2}\cdot\left[y_S(\theta) + y_S^*(\theta)\right] \qquad \mathrm{Im}[y_S(\theta)] = \frac{1}{2j}\cdot\left[y_S(\theta) - y_S^*(\theta)\right]$$

$$\mathrm{Re}[y_S(\theta)] = \frac{1}{2}\cdot\left[\frac{1-\Gamma_S\cdot e^{2j\theta}}{1+\Gamma_S\cdot e^{2j\theta}} + \frac{1-\Gamma_S^*\cdot e^{-2j\theta}}{1+\Gamma_S^*\cdot e^{-2j\theta}}\right] \qquad \Gamma_S = |\Gamma_S|\cdot e^{j\varphi}$$

$$\mathrm{Re}[y_S(\theta)] = \frac{1}{2}\cdot\left[\frac{\left(1-|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)\cdot\left(1+|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right) + \left(1-|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1+|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}{\left(1+|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1+|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}\right]$$

$$\mathrm{Re}[y_S(\theta)] = \frac{1}{2}\cdot\left[\frac{2-2\cdot|\Gamma_S|^2}{1+|\Gamma_S|^2+2\cdot|\Gamma_S|\cdot\cos(\varphi+2\theta)}\right] = 1 \quad \Rightarrow \quad \boxed{\cos(\varphi+2\theta) = -|\Gamma_S|}$$

A.S., Γ, series line, usage

- Equations for computing the series line θ:

$$\mathrm{Re}[y_S(\theta)] = 1 \Rightarrow \qquad \boxed{\cos(\varphi + 2\theta) = -|\Gamma_S|}$$

$$\Gamma_S = |\Gamma_S| \cdot e^{j\varphi} \qquad \Gamma_S = 0.593\angle 46.85° \qquad |\Gamma_S| = 0.593; \quad \varphi = 46.85°$$

- two solutions possible, in the 0÷180° range
  - add λ/2 ⇔ 180° as needed

$$\theta = \frac{1}{2} \cdot \left[\pm\cos^{-1}(-|\Gamma_S|) - \varphi + k \cdot 360°\right] = \frac{1}{2} \cdot \left[\pm\cos^{-1}(-|\Gamma_S|) - \varphi\right] + k \cdot 180°$$

$$\forall k \in N$$

$$\cos(\varphi + 2\theta) = -0.593 \Rightarrow \qquad (\varphi + 2\theta) = \pm 126.35°$$

$$(46.85° + 2\theta) = \begin{cases} +126.35° \\ -126.35° \end{cases} \qquad \theta = \begin{cases} +39.7° \\ -86.6° + 180° = +93.4° \end{cases}$$

A.S., Γ, shunt stub, proof

- Equations for computing the shunt stub θ_{sp}:

$$\mathrm{Re}[y_S(\theta)] = 1 \qquad \cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\mathrm{Im}[y_S(\theta)] = \frac{1}{2j} \cdot \left[\frac{1-\Gamma_S \cdot e^{2j\theta}}{1+\Gamma_S \cdot e^{2j\theta}} - \frac{1-\Gamma_S^* \cdot e^{-2j\theta}}{1+\Gamma_S^* \cdot e^{-2j\theta}} \right] \qquad \Gamma_S = |\Gamma_S| \cdot e^{j\varphi}$$

$$\mathrm{Im}[y_S(\theta)] = \frac{1}{2j} \cdot \left[\frac{\left(1-|\Gamma_S| \cdot e^{j\cdot(\varphi+2\theta)}\right) \cdot \left(1+|\Gamma_S| \cdot e^{-j\cdot(\varphi+2\theta)}\right) - \left(1-|\Gamma_S| \cdot e^{-j\cdot(\varphi+2\theta)}\right) \cdot \left(1+|\Gamma_S| \cdot e^{j\cdot(\varphi+2\theta)}\right)}{\left(1+|\Gamma_S| \cdot e^{-j\cdot(\varphi+2\theta)}\right) \cdot \left(1+|\Gamma_S| \cdot e^{j\cdot(\varphi+2\theta)}\right)} \right]$$

$$\mathrm{Im}[y_S(\theta)] = \frac{1}{2j} \cdot \left[\frac{2 \cdot |\Gamma_S| \cdot e^{-j\cdot(\varphi+2\theta)} - 2 \cdot |\Gamma_S| \cdot e^{+j\cdot(\varphi+2\theta)}}{1+|\Gamma_S|^2 + 2 \cdot |\Gamma_S| \cdot \cos(\varphi+2\theta)} \right] = \frac{-2 \cdot |\Gamma_S| \cdot \sin(\varphi+2\theta)}{1+|\Gamma_S|^2 + 2 \cdot |\Gamma_S| \cdot \cos(\varphi+2\theta)}$$

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \Rightarrow \qquad \mathrm{Im}[y_S(\theta)] = \frac{-2 \cdot |\Gamma_S| \cdot \sin(\varphi+2\theta)}{1-|\Gamma_S|^2}$$

A.S., Γ, shunt stub, proof

- Equations for computing the shunt stub

$$\cos(\varphi+2\theta)=-|\Gamma_S| \Rightarrow \qquad \sin(\varphi+2\theta)=\pm\sqrt{1-|\Gamma_S|^2}$$

$$\mathrm{Im}[y_S(\theta)]=\frac{-2\cdot|\Gamma_S|\cdot\sin(\varphi+2\theta)}{1-|\Gamma_S|^2} \Rightarrow \qquad \mathrm{Im}[y_S(\theta)]=\frac{\mp 2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}$$

- two cases

$$\varphi+2\theta\in[0°,180°]\Rightarrow \sin(\varphi+2\theta)\geq 0 \quad \begin{cases} \sin(\varphi+2\theta)=\sqrt{1-|\Gamma_S|^2} \\ \mathrm{Im}[y_S(\theta)]=\dfrac{-2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}} \end{cases}$$

$$\varphi+2\theta\in(-180°,0°)\Rightarrow \sin(\varphi+2\theta)< 0 \quad \begin{cases} \sin(\varphi+2\theta)=-\sqrt{1-|\Gamma_S|^2} \\ \mathrm{Im}[y_S(\theta)]=\dfrac{2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}} \end{cases}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

A.S., Γ, shunt stub, proof

- We prefer (for microstrip) open circuited stub

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- The normalized susceptance to be introduced to achieve the match
  - Y(θ) is the admittance seen **towards** the source, Zo parallel with j·B

$$b = \mathrm{Im}\left[\frac{Y_{in,oc}}{Y_0}\right] = \mathrm{Im}\left[\frac{Z_0}{Z_{in,oc}}\right] = \tan \beta \cdot l = \mathrm{Im}[y_S(\theta)]$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

Analytical solution, Γ, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \operatorname{Im}[y_S(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

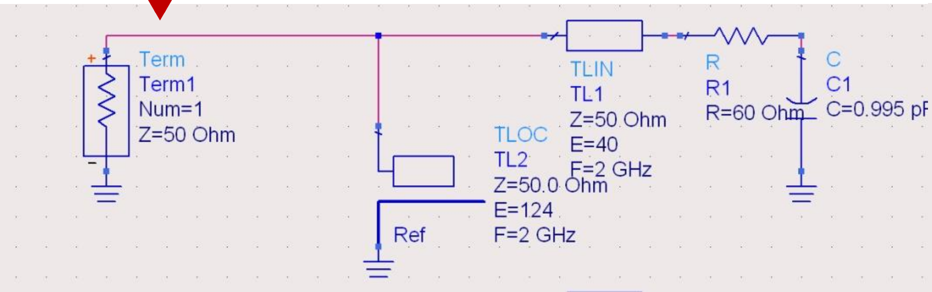
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

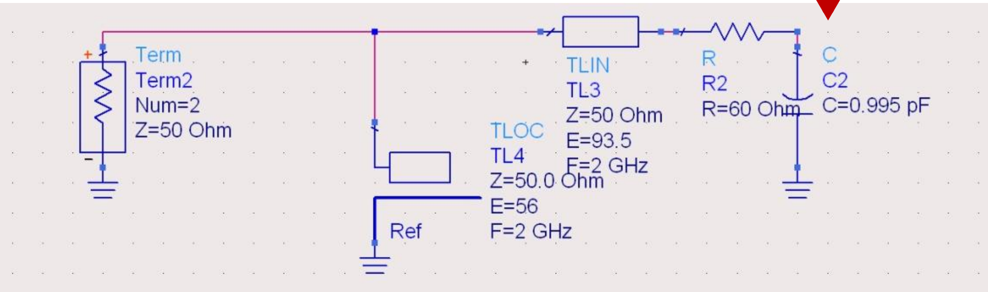
$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$



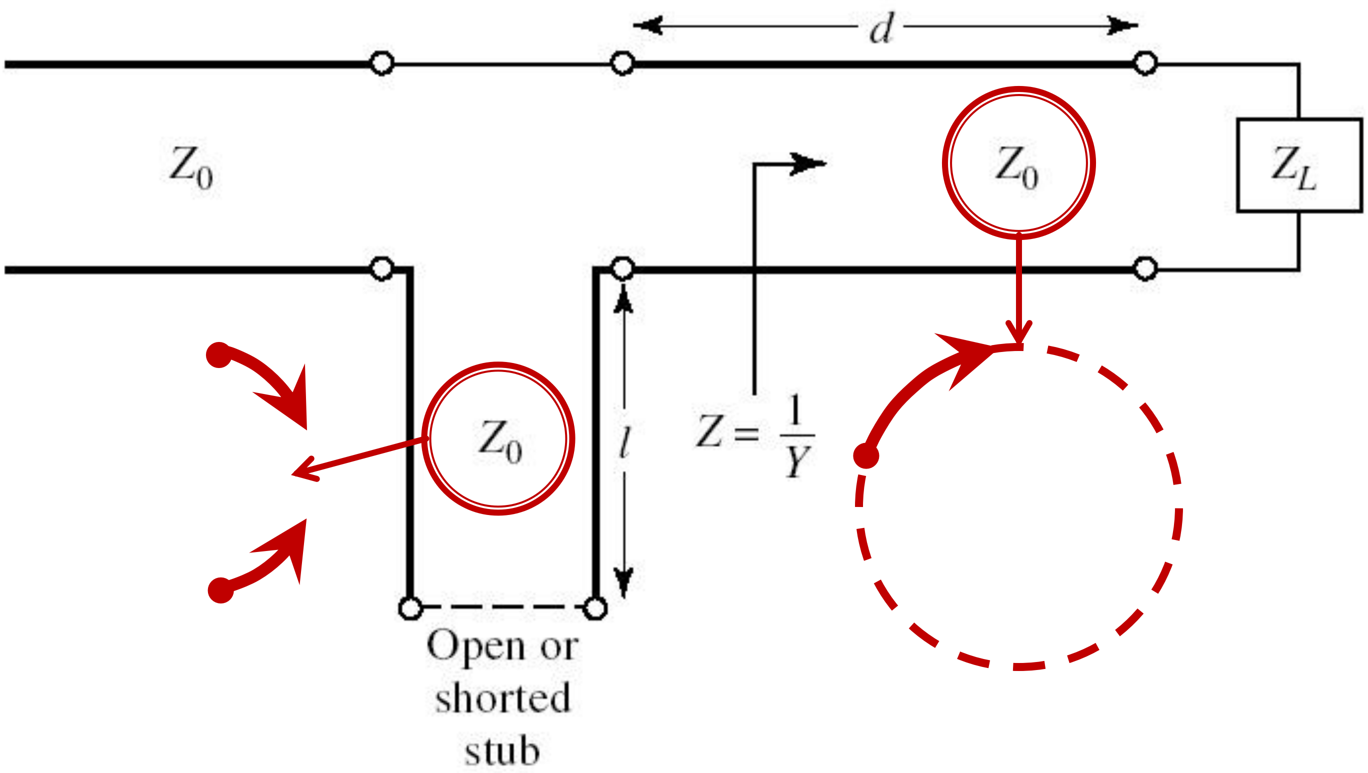


Series Stub

Sectiune de linie serie

Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



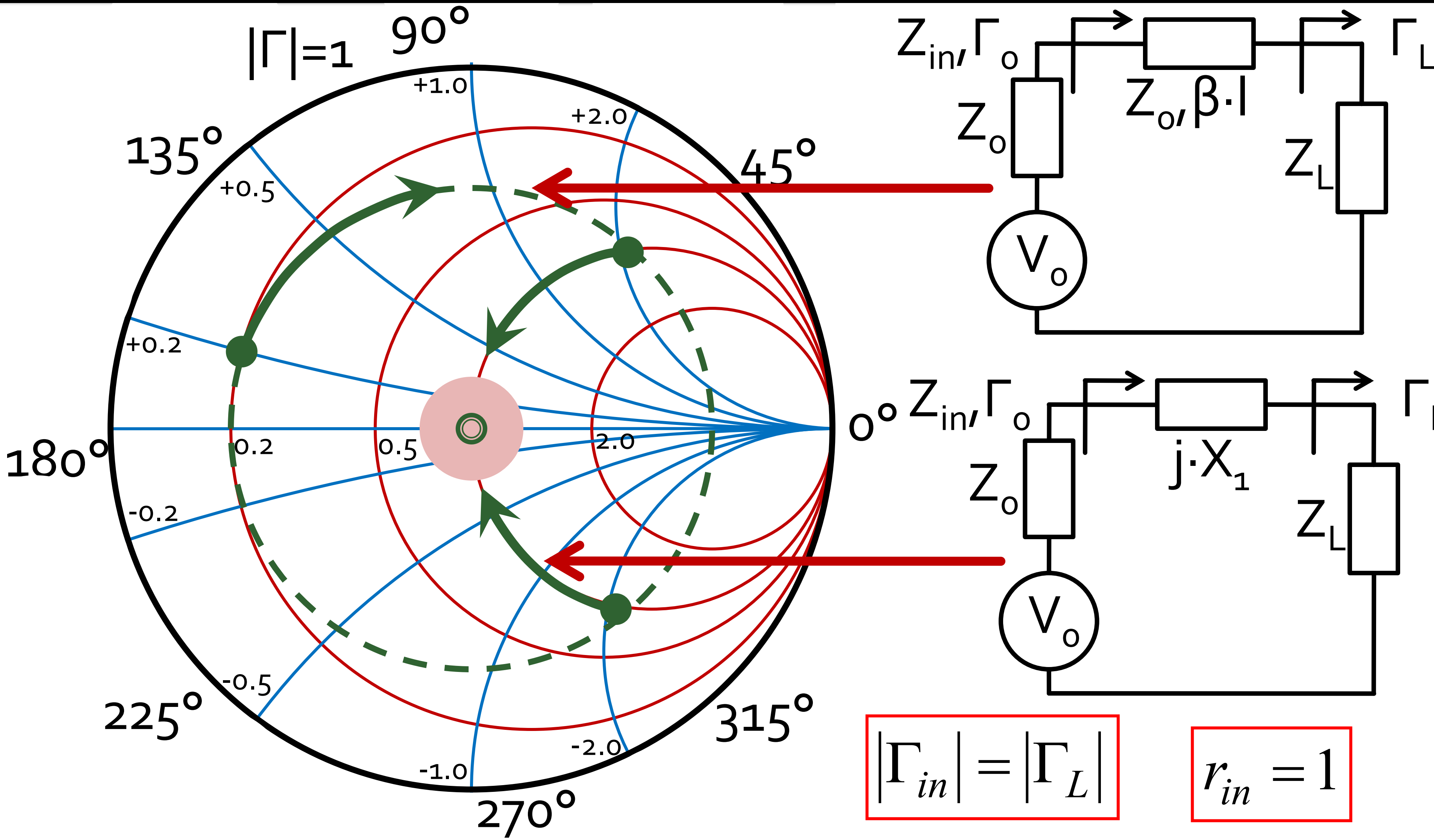
Case 2, Series Stub

- We use a series transmission line to move the reflection coefficient **on the circle** $r_L = 1$
- We compensate the remaining reactive part of the load with a series reactance to achieve match
- The series reactance is made with a stub which can be, as needed:
  - open-circuited
  - short-circuited

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

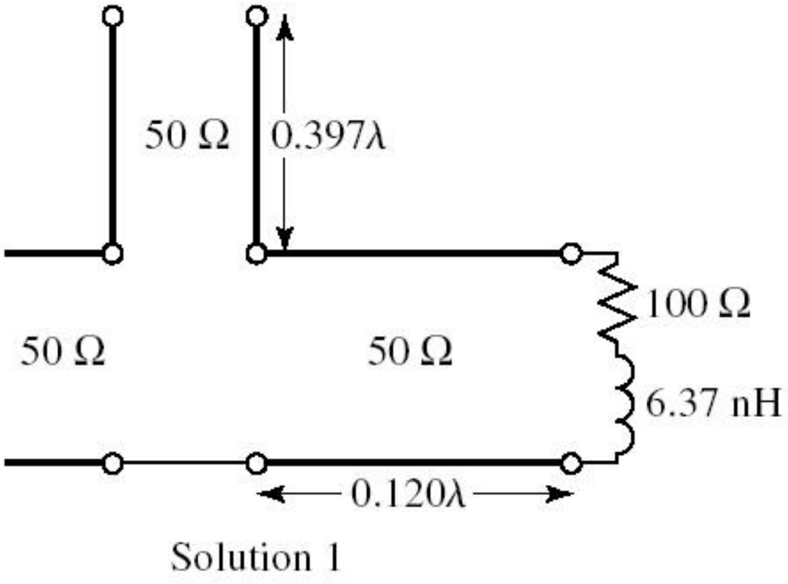
$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \qquad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

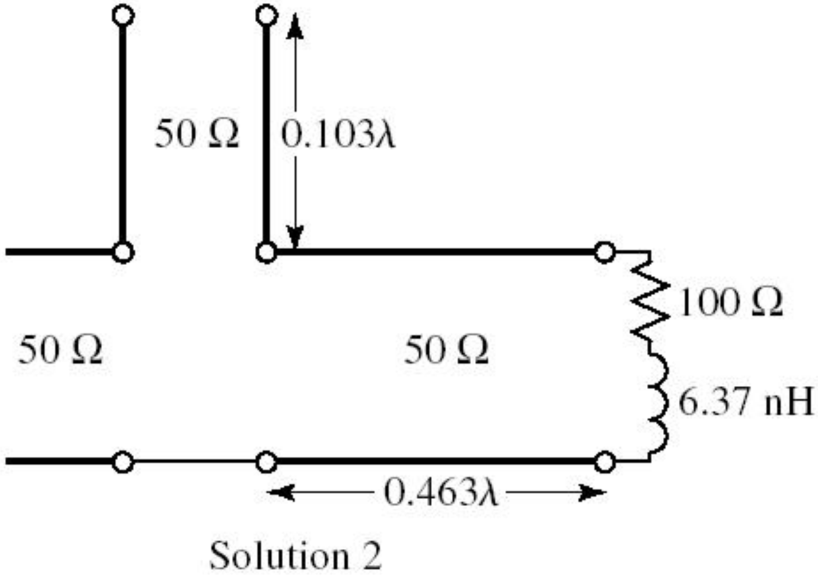
Matching, series line + series reactance



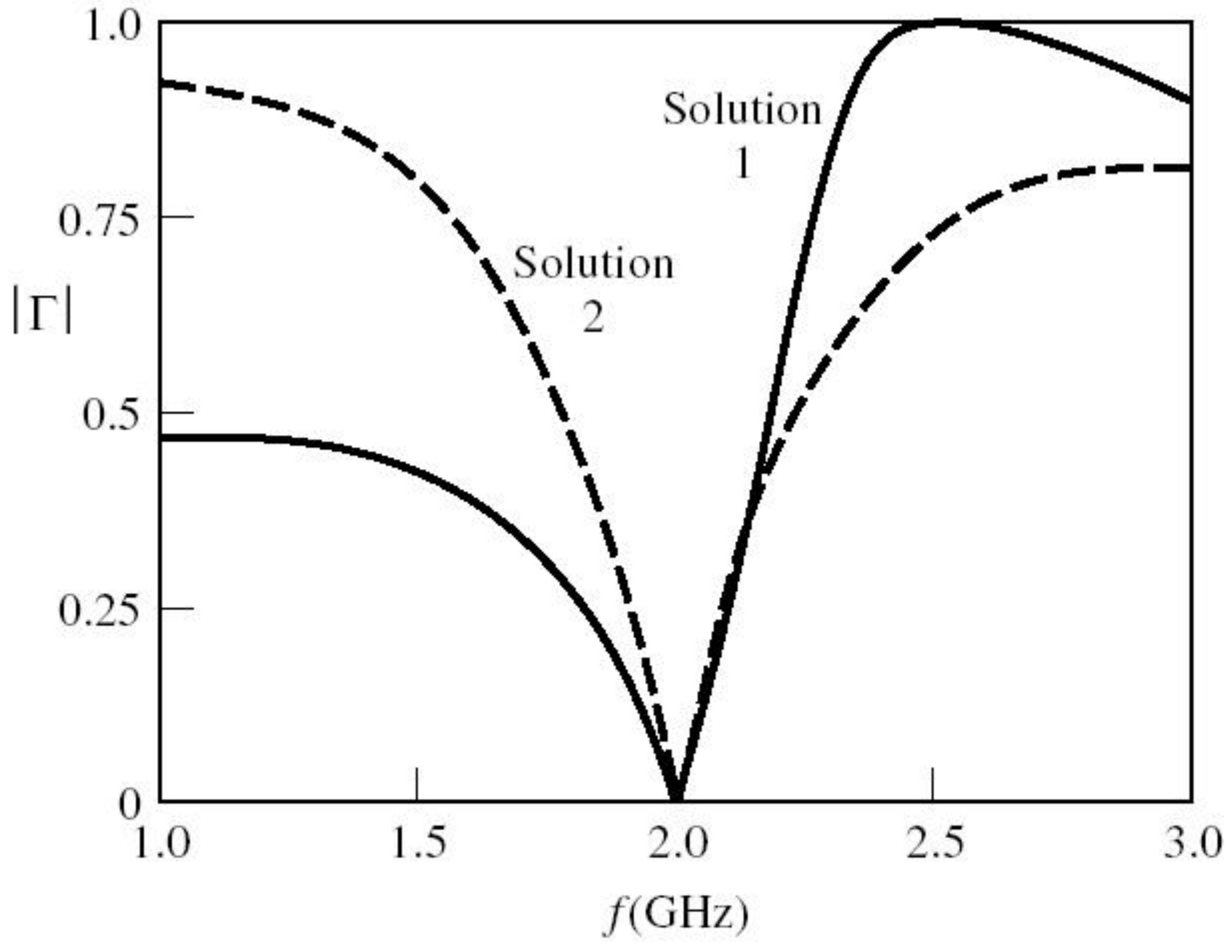
Example, Series Stub, oc.

- load: 100 Ω series with 6.37 nH at 2 GHz
- two solutions possible

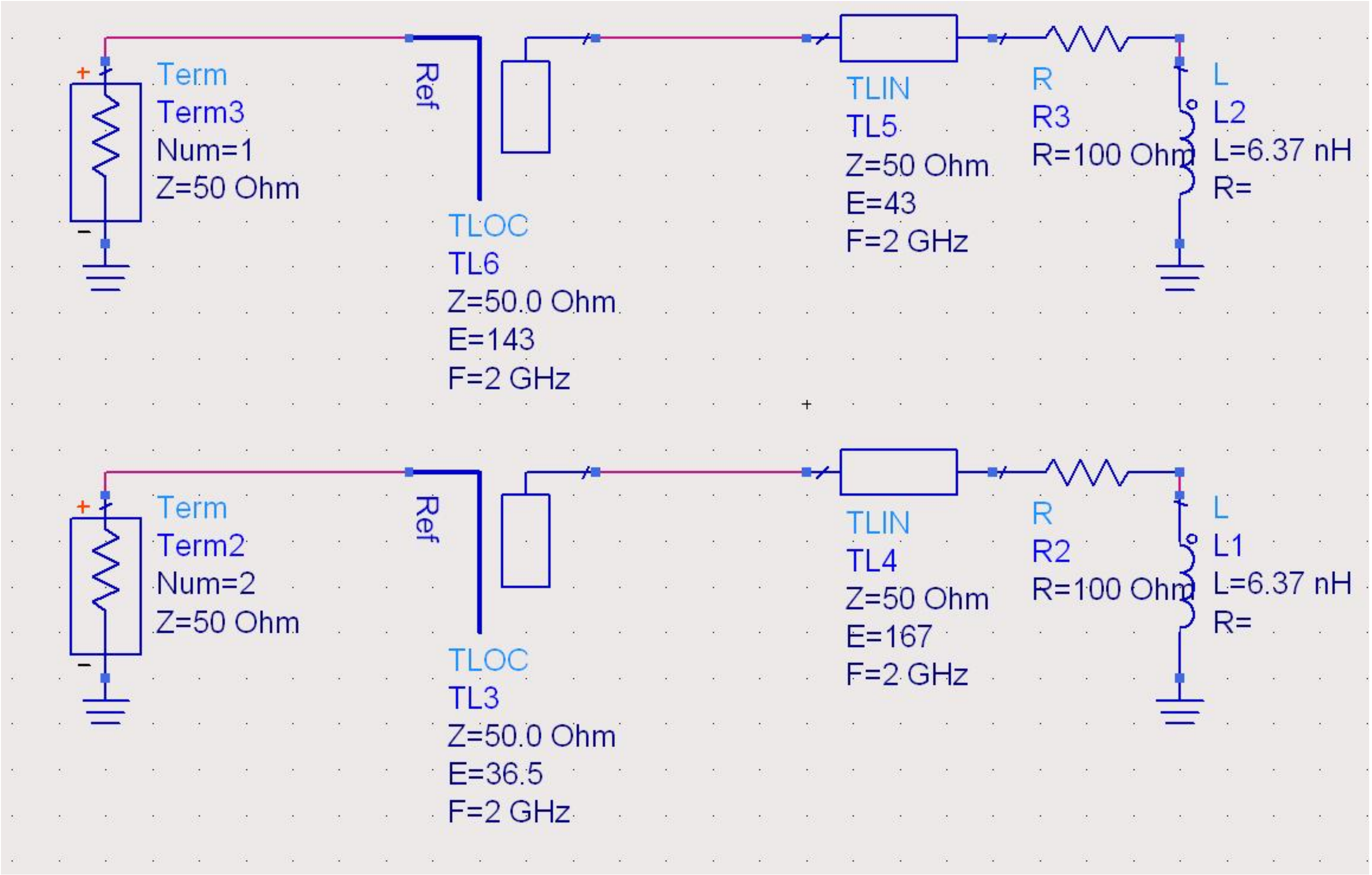




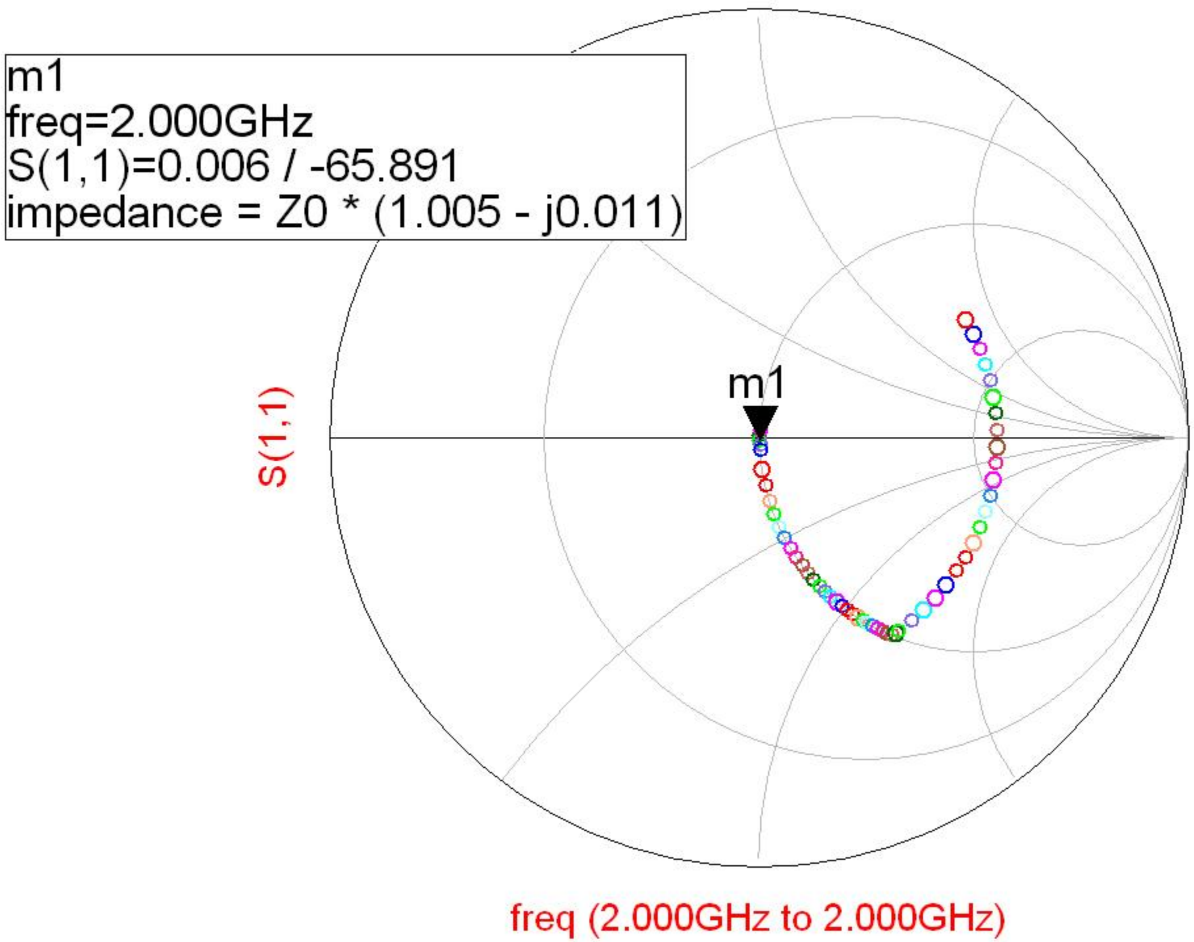
Example, Series Stub, oc.



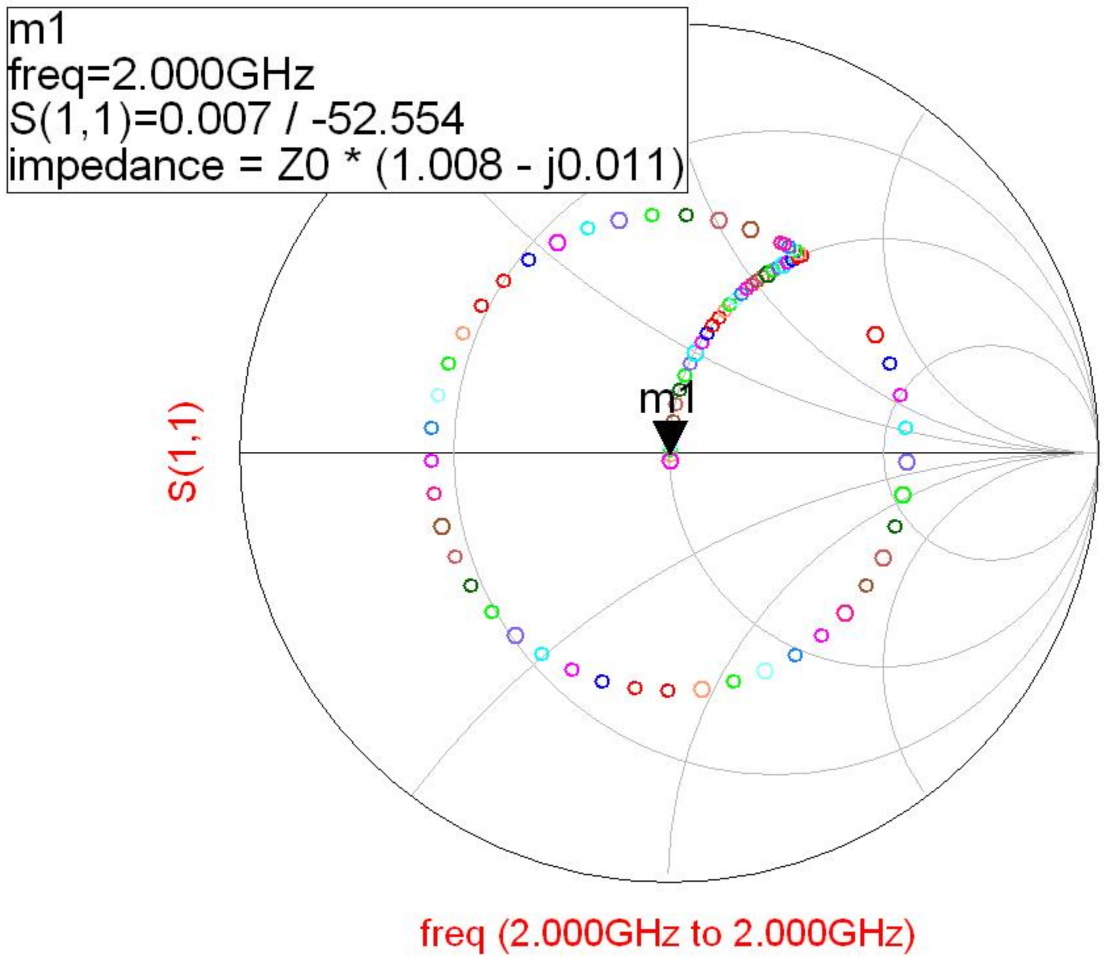
Example, Series Stub, oc.



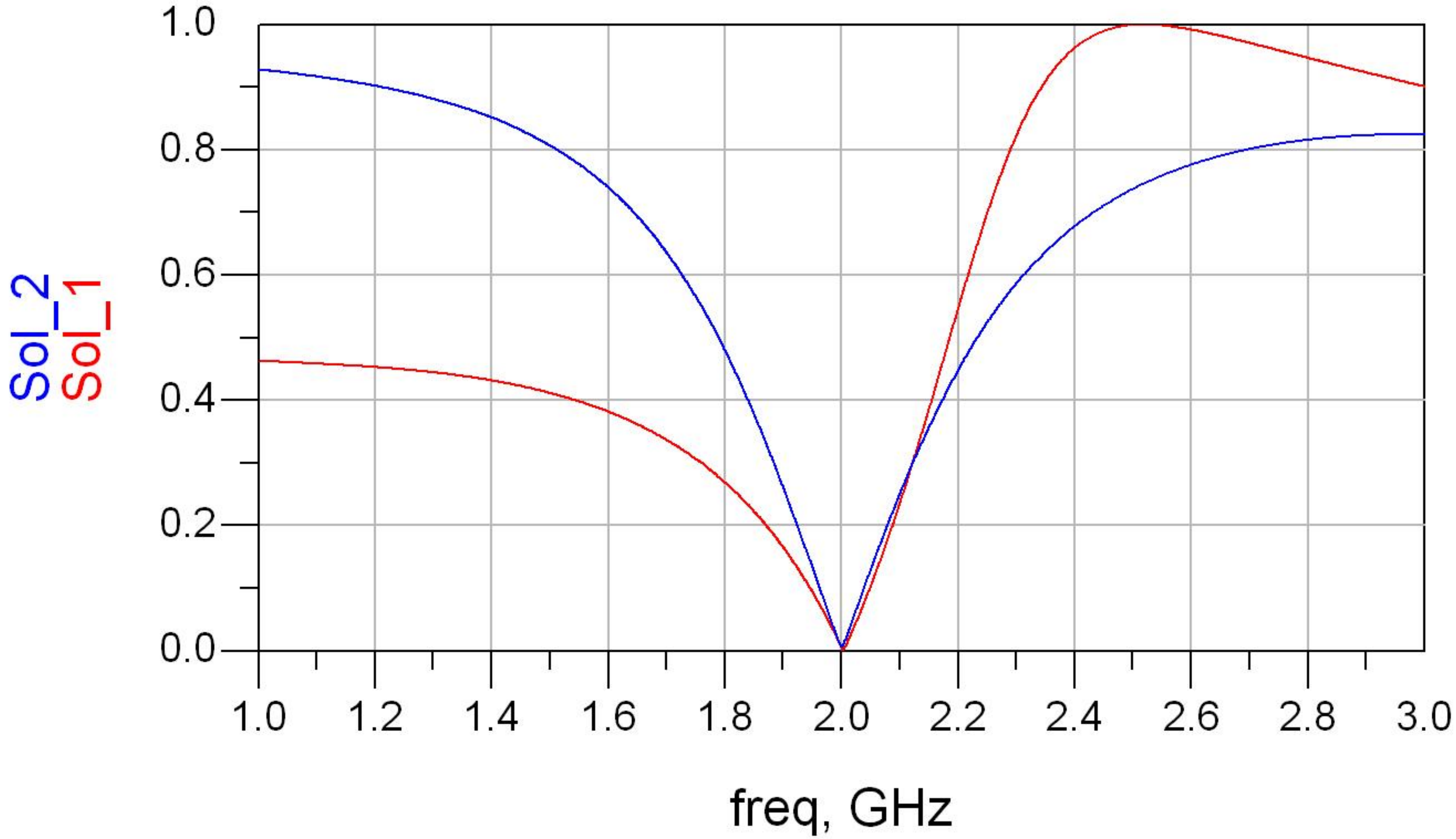
Example, Series Stub, oc.



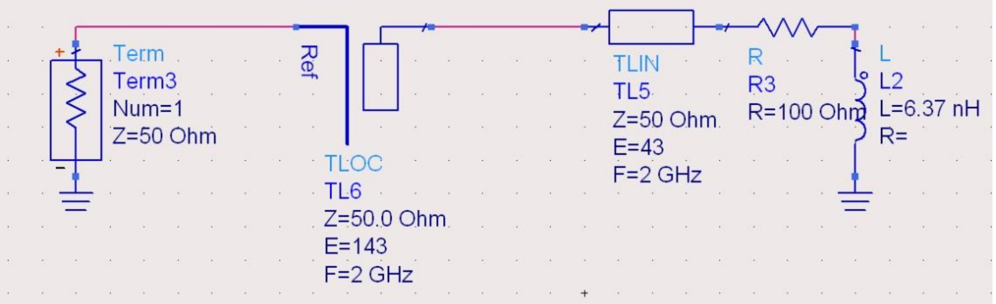
Example, Series Stub, oc.



Example, Series Stub, oc.



Example, Series Stub, oc.

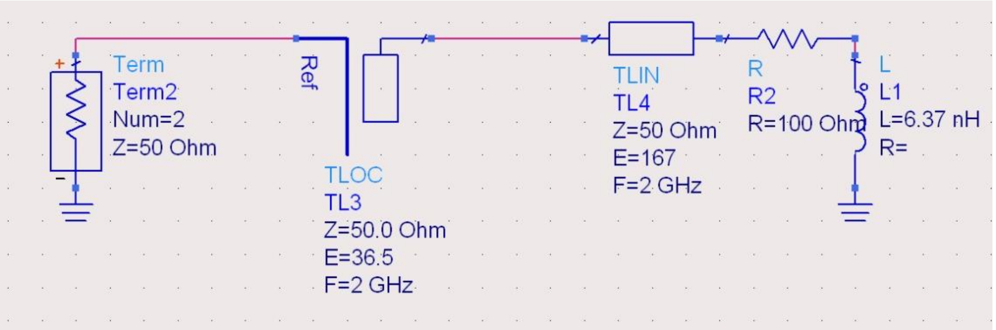


$$l_1 = \frac{43^\circ}{360^\circ} \cdot \lambda = 0.119 \cdot \lambda$$

$$l_2 = \frac{143^\circ}{360^\circ} \cdot \lambda = 0.397 \cdot \lambda$$

$$l_1 = \frac{167^\circ}{360^\circ} \cdot \lambda = 0.464 \cdot \lambda$$

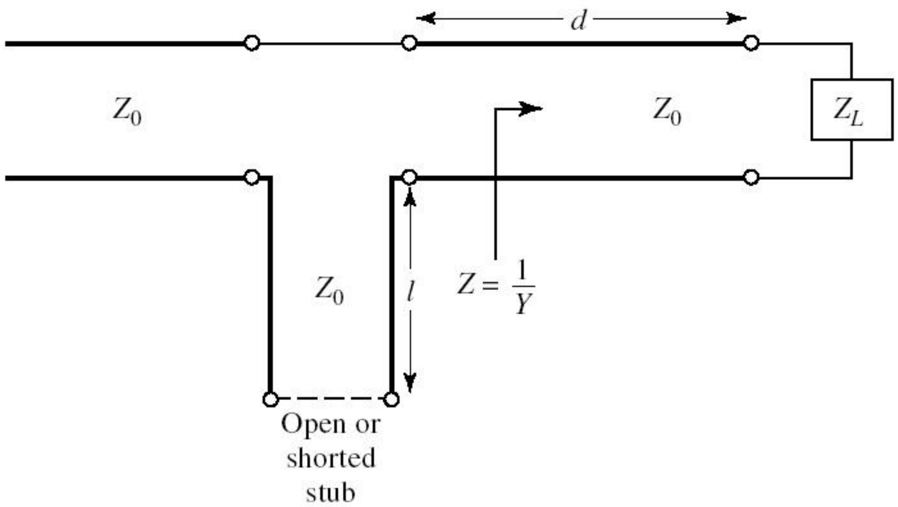
$$l_2 = \frac{36.5^\circ}{360^\circ} \cdot \lambda = 0.101 \cdot \lambda$$



Analytical solution

Series Stub

Analytical solution, impedances



$$Y_L = \frac{1}{Z_L} = G_L + j \cdot B_L$$

$$Y = Y_0 \cdot \frac{(G_L + j \cdot B_L) + j \cdot Y_0 \cdot t}{Y_0 + j \cdot (G_L + j \cdot B_L) \cdot t}$$

$$t \overset{not}{=} \tan \beta \cdot d \qquad Z = R + j \cdot X = \frac{1}{Y}$$

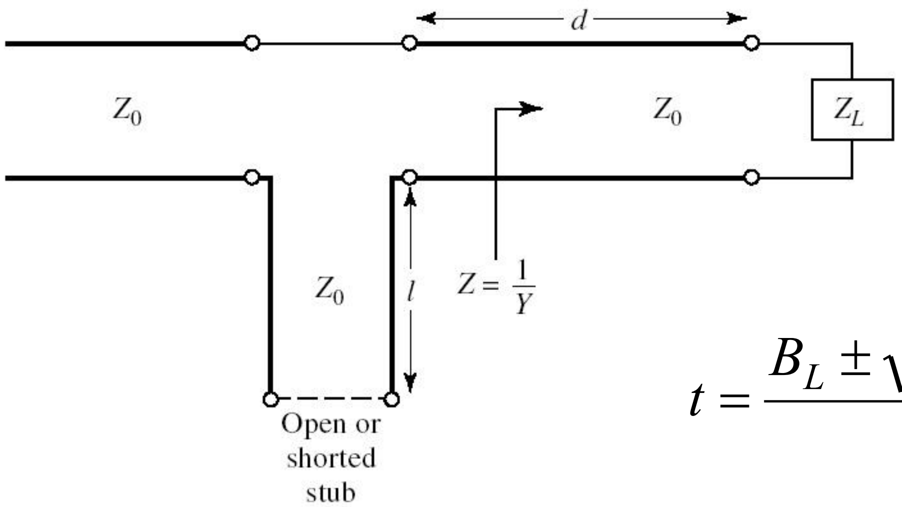
$$R = \frac{G_L \cdot (1 + t^2)}{G_L^2 + (G_L + Y_0 \cdot t)^2}$$

$$X = \frac{G_L^2 \cdot t - (Y_0 - B_L \cdot t) \cdot (B_L + Y_0 \cdot t)}{Y_0 \cdot \left[G_L^2 + (B_L + Y_0 \cdot t)^2\right]}$$

- *d* (which implies t) is chosen so that: $R = Z_0 = \frac{1}{Y_0}$

$$Y_0 \cdot (G_L - Y_0) \cdot t^2 - 2 \cdot B_L \cdot Y_0 \cdot t + (G_L \cdot Y_0 - G_L^2 - B_L^2) = 0$$

Analytical solution



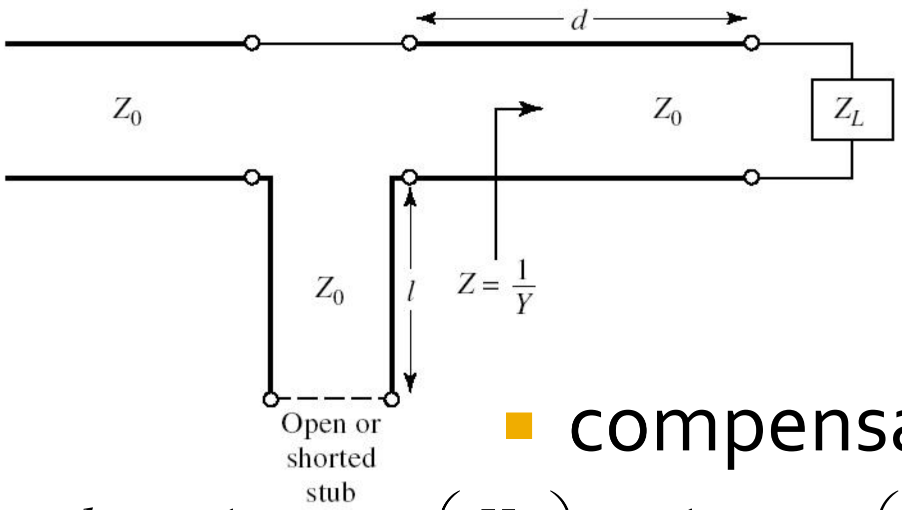
$$t = \frac{-B_L}{2 \cdot Y_0}, \quad G_L = Y_0$$

$$t = \frac{B_L \pm \sqrt{G_L \cdot \left[(Y_0 - G_L)^2 + B_L^2 \right] / Y_0}}{G_L - Y_0} \quad G_L \neq Y_0$$

- second grade equation, 2 solutions possible
- *d* computed from *t*

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi} \cdot \arctan t & t \geq 0 \\ \frac{1}{2\pi} \cdot (\pi + \arctan t) & t < 0 \end{cases}$$

Analytical solution



$$X_S = -X$$

$$X = \frac{G_L^2 \cdot t - (Y_0 - B_L \cdot t)\cdot(B_L + Y_0 \cdot t)}{Y_0 \cdot \left[G_L^2 + (B_L + Y_0 \cdot t)^2\right]}$$

- compensating reactance is:

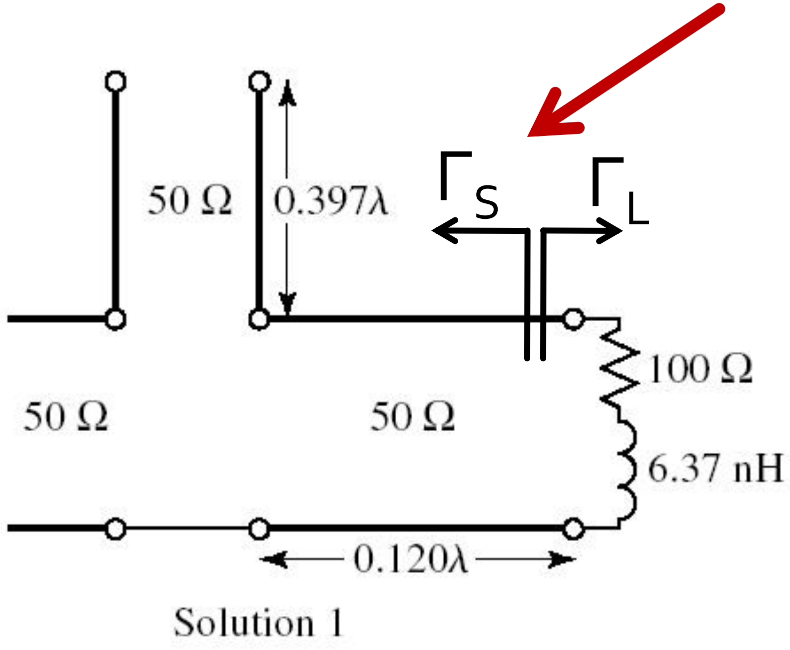
$$\frac{l_{sc}}{\lambda} = \frac{1}{2\pi} \cdot \arctan\left(\frac{X_S}{Z_0}\right) = \frac{-1}{2\pi} \cdot \arctan\left(\frac{X}{Z_0}\right)$$

$$\frac{l_{oc}}{\lambda} = \frac{-1}{2\pi} \cdot \arctan\left(\frac{Z_0}{X_S}\right) = \frac{1}{2\pi} \cdot \arctan\left(\frac{Z_0}{X}\right)$$

- for **negative lengths** we add λ/2

Analytical solution, reflection coefficient

- load: 100 Ω series with 6.37 nH at 2 GHz



$$Z_L = R_L + \frac{1}{j \cdot \omega \cdot C_L} = 100\Omega + j \cdot 80.05\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.481 + j \cdot 0.277$$

$$z_L = \frac{Z_L}{Z_0} = 2 + j \cdot 1.6$$

- matching requires obtaining conjugate value for Γ

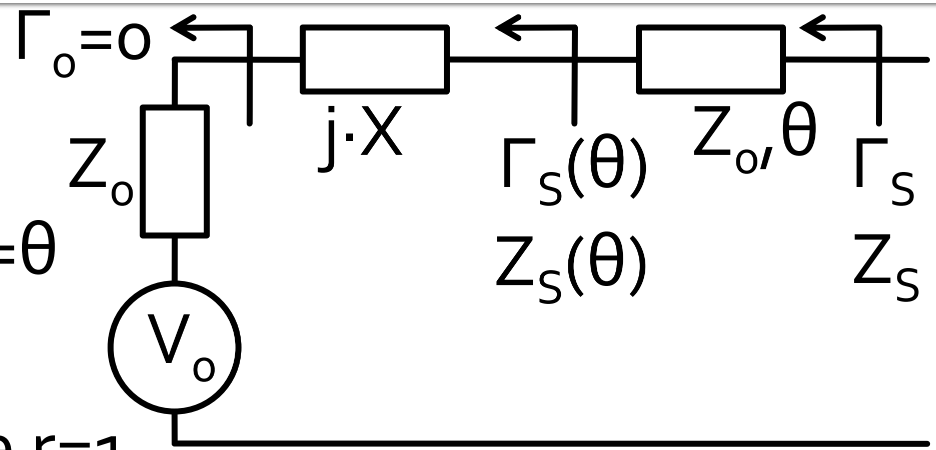
$$\Gamma_S = \Gamma_L^* = 0.481 - j \cdot 0.277$$

$$\Gamma_S = 0.555\angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92°$$

Analytical solution, Γ

- series line
  - electrical length E= β·l=θ
  - moves the reflection coefficient on the circle r=1
- series stub
  - electrical length E= $\beta \cdot l_{ss} = \theta_{ss}$
  - moves the reflection coefficient to the center of the Smith Chart (Γ_o=0)



$$z_S = \frac{Z_S}{Z_0} = \frac{Z_S}{50\Omega}$$

$$z_S = \frac{1+\Gamma_S}{1-\Gamma_S} = 2 - j \cdot 1.6$$

$$\Gamma_S(\theta) = \Gamma_S \cdot e^{2j\theta}$$

$$z_S(\theta) = \frac{1+\Gamma_S(\theta)}{1-\Gamma_S(\theta)} = \frac{1+\Gamma_S \cdot e^{2j\theta}}{1-\Gamma_S \cdot e^{2j\theta}}$$

A.S., Γ, series line, proof

- After the series line with electrical length θ:

$$\mathrm{Re}[z_S(\theta)] = 1 \qquad \mathrm{Im}[z_S(\theta)] = X$$

$$\mathrm{Re}[z_S(\theta)] = \frac{1}{2}\cdot\left[z_S(\theta) + z_S^*(\theta)\right] \qquad \mathrm{Im}[z_S(\theta)] = \frac{1}{2j}\cdot\left[z_S(\theta) - z_S^*(\theta)\right]$$

$$\mathrm{Re}[z_S(\theta)] = \frac{1}{2}\cdot\left[\frac{1+\Gamma_S\cdot e^{2j\theta}}{1-\Gamma_S\cdot e^{2j\theta}} + \frac{1+\Gamma_S^*\cdot e^{-2j\theta}}{1-\Gamma_S^*\cdot e^{-2j\theta}}\right] \qquad \Gamma_S = |\Gamma_S|\cdot e^{j\varphi}$$

$$\mathrm{Re}[z_S(\theta)] = \frac{1}{2}\cdot\left[\frac{\left(1+|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)+\left(1+|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}{\left(1-|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}\right]$$

$$\mathrm{Re}[z_S(\theta)] = \frac{1}{2}\cdot\left[\frac{2-2\cdot|\Gamma_S|^2}{1+|\Gamma_S|^2-2\cdot|\Gamma_S|\cdot\cos(\varphi+2\theta)}\right] = 1 \quad \Rightarrow \quad \boxed{\cos(\varphi+2\theta) = |\Gamma_S|}$$

A.S., Γ, series line. usage

- Equations for computing the series line θ:

$$\mathrm{Re}[z_S(\theta)] = 1 \Rightarrow \qquad \boxed{\cos(\varphi + 2\theta) = |\Gamma_S|}$$

$$\Gamma_S = |\Gamma_S| \cdot e^{j\varphi} \qquad \Gamma_S = 0.555\angle -29.92^\circ \qquad |\Gamma_S| = 0.555; \quad \varphi = -29.92^\circ$$

- two solutions possible, in the 0÷180° range (add λ/2 ⇔ 180° as needed)

$$\theta = \frac{1}{2} \cdot \left[\pm \cos^{-1}(|\Gamma_S|) - \varphi + k \cdot 360^\circ\right] = \frac{1}{2} \cdot \left[\pm \cos^{-1}(|\Gamma_S|) - \varphi\right] + k \cdot 180^\circ$$

$$\forall k \in N$$

$$\cos(\varphi + 2\theta) = 0.555 \Rightarrow \qquad (\varphi + 2\theta) = \pm 56.28^\circ$$

$$(-29.92^\circ + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \qquad \theta = \begin{cases} +43.1^\circ \\ -13.2^\circ + 180^\circ = +166.8^\circ \end{cases}$$

A.S., Γ, series stub, proof

- Equations for computing the series stub θ_{ss}:

$$\mathrm{Re}[z_S(\theta)] = 1 \qquad \cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\mathrm{Im}[z_S(\theta)] = \frac{1}{2j}\cdot\left[\frac{1+\Gamma_S\cdot e^{2j\theta}}{1-\Gamma_S\cdot e^{2j\theta}} - \frac{1+\Gamma_S^*\cdot e^{-2j\theta}}{1-\Gamma_S^*\cdot e^{-2j\theta}}\right] \qquad \Gamma_S = |\Gamma_S|\cdot e^{j\varphi}$$

$$\mathrm{Im}[z_S(\theta)] = \frac{1}{2j}\cdot\left[\frac{\left(1+|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right) - \left(1+|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}{\left(1-|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}\right)\cdot\left(1-|\Gamma_S|\cdot e^{j\cdot(\varphi+2\theta)}\right)}\right]$$

$$\mathrm{Im}[z_S(\theta)] = \frac{1}{2j}\cdot\left[\frac{2\cdot|\Gamma_S|\cdot e^{+j\cdot(\varphi+2\theta)} - 2\cdot|\Gamma_S|\cdot e^{-j\cdot(\varphi+2\theta)}}{1+|\Gamma_S|^2 - 2\cdot|\Gamma_S|\cdot\cos(\varphi+2\theta)}\right] = \frac{2\cdot|\Gamma_S|\cdot\sin(\varphi+2\theta)}{1+|\Gamma_S|^2 - 2\cdot|\Gamma_S|\cdot\cos(\varphi+2\theta)}$$

$$\cos(\varphi+2\theta) = |\Gamma_S| \Rightarrow \qquad \mathrm{Im}[z_S(\theta)] = \frac{2\cdot|\Gamma_S|\cdot\sin(\varphi+2\theta)}{1-|\Gamma_S|^2}$$

A.S., Γ, series stub, proof

- Equations for computing the series stub θ_{ss}:

$$\cos(\varphi+2\theta)=|\Gamma_S| \Rightarrow \quad \sin(\varphi+2\theta)=\pm\sqrt{1-|\Gamma_S|^2}$$

$$\mathrm{Im}[z_S(\theta)]=\frac{2\cdot|\Gamma_S|\cdot\sin(\varphi+2\theta)}{1-|\Gamma_S|^2} \Rightarrow \quad \mathrm{Im}[z_S(\theta)]=\frac{\pm 2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}$$

- two cases:

$$\varphi+2\theta\in[0°,180°]\Rightarrow \sin(\varphi+2\theta)\geq 0 \quad \begin{cases} \sin(\varphi+2\theta)=\sqrt{1-|\Gamma_S|^2} \\ \mathrm{Im}[z_S(\theta)]=\dfrac{2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}} \end{cases}$$

$$\varphi+2\theta\in(-180°,0°)\Rightarrow \sin(\varphi+2\theta)< 0 \quad \begin{cases} \sin(\varphi+2\theta)=-\sqrt{1-|\Gamma_S|^2} \\ \mathrm{Im}[z_S(\theta)]=\dfrac{-2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}} \end{cases}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

A.S., Γ, series stub, proof

- We prefer (for microstrip) open circuited stub

$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- The normalized reactance to be introduced to achieve the match
  - Z(θ) is the impedance seen **towards** the source, Zo series with j·X

$$x = \mathrm{Im}\left[\frac{Z_{in,oc}}{Z_0}\right] = -\cot \beta \cdot l = \mathrm{Im}[z_S(\theta)]$$

$$\boxed{\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}}$$

Analytical solution, Γ, usage

$$(\varphi+2\theta)=\begin{cases}+56.28^\circ\\-56.28^\circ\end{cases}\quad \theta=\begin{cases}43.1^\circ\\166.8^\circ\end{cases}\quad \mathrm{Im}[z_S(\theta)]=\begin{cases}+1.335\\-1.335\end{cases}\quad \theta_{ss}=\begin{cases}-36.8^\circ+180^\circ=143.2^\circ\\+36.8^\circ\end{cases}$$

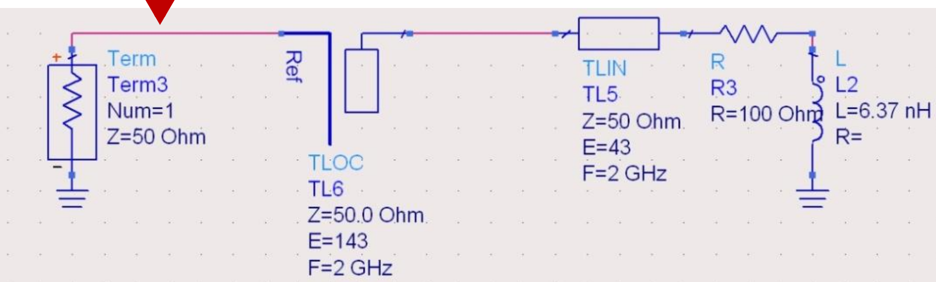
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

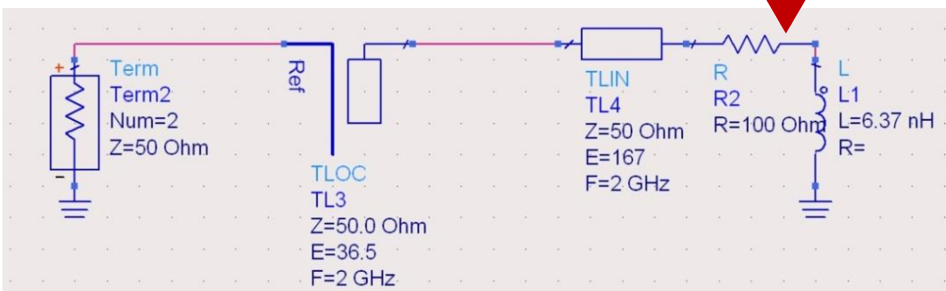
$$l_1=\frac{43.1^\circ}{360^\circ}\cdot\lambda=0.120\cdot\lambda$$

$$l_2=\frac{143.2^\circ}{360^\circ}\cdot\lambda=0.398\cdot\lambda$$

$$l_1=\frac{166.8^\circ}{360^\circ}\cdot\lambda=0.463\cdot\lambda$$

$$l_2=\frac{36.8^\circ}{360^\circ}\cdot\lambda=0.102\cdot\lambda$$



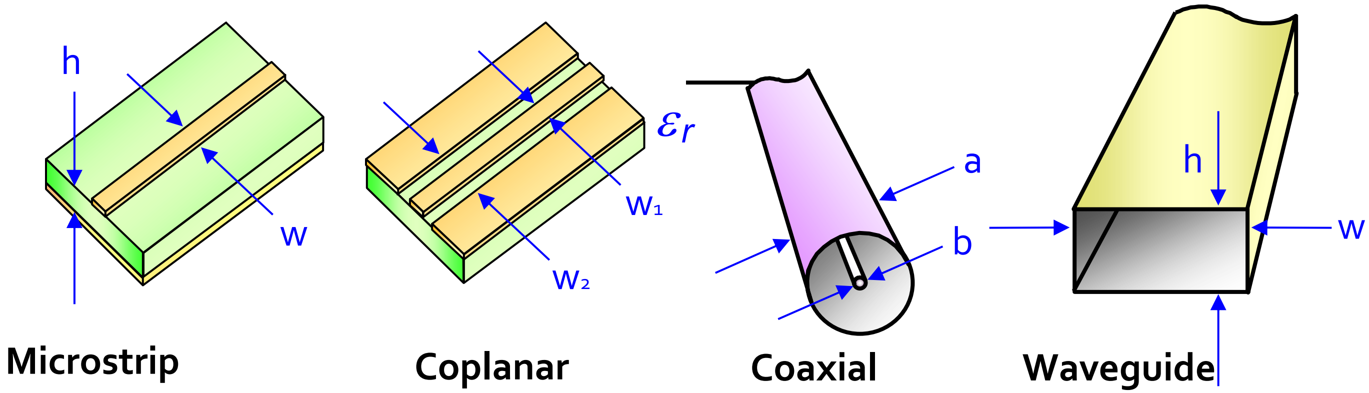


Single stub tuning

- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account:
  - physical dimensions (area occupied on chip/board)
  - sensitivity of the match on length error ($\Delta\Gamma/\Delta E$, $\Delta\Gamma/\Delta l$)
  - convenient frequency behavior (bandwidth)

Single stub tuning

- We choose one of the 8 possible solutions (series/shunt, oc./sc.), taking into account :
  - physical realizability (in the line technology we use)



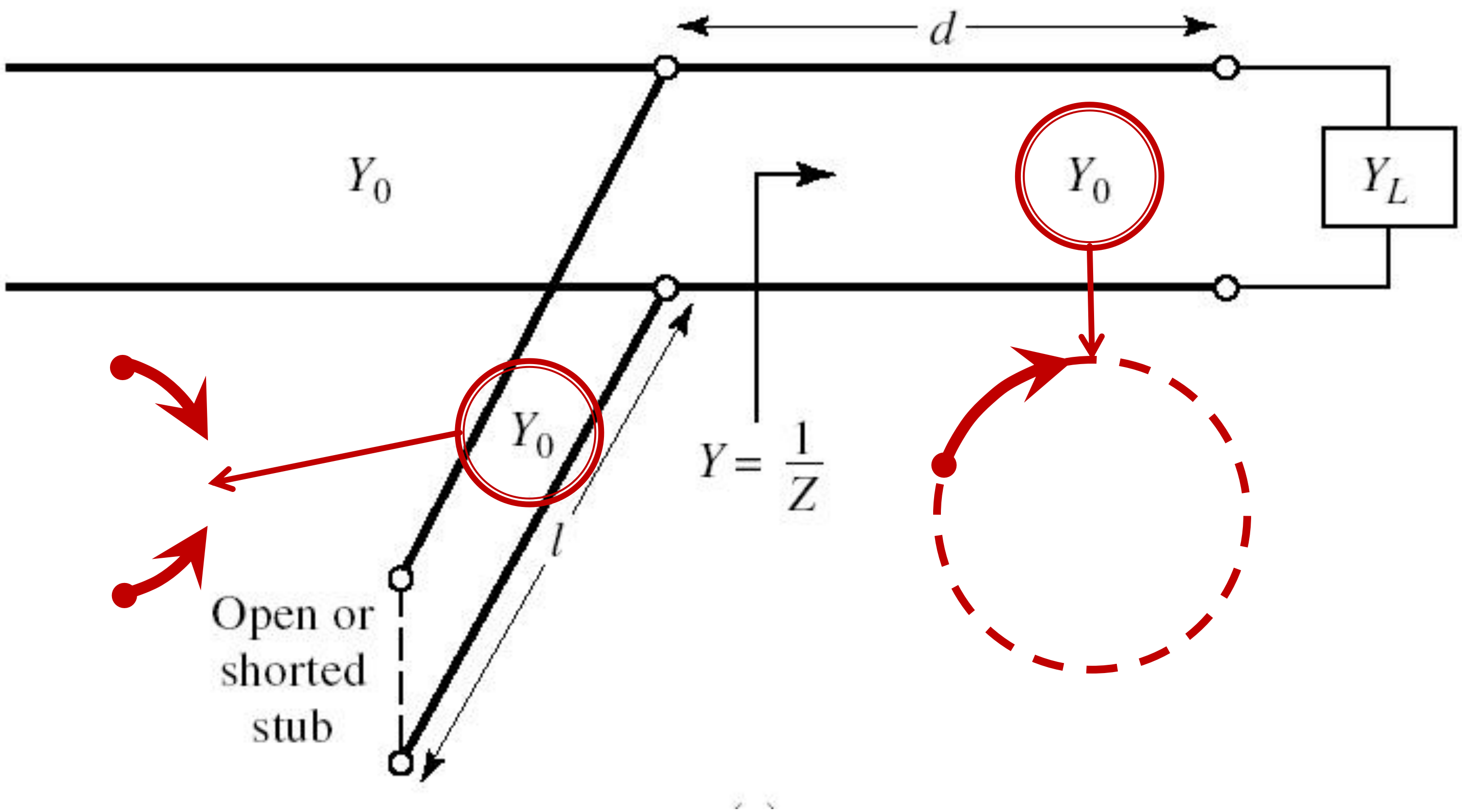
- Main disadvantage:
  - requires a **variable length of line** between the load and the stub

Analytical solutions

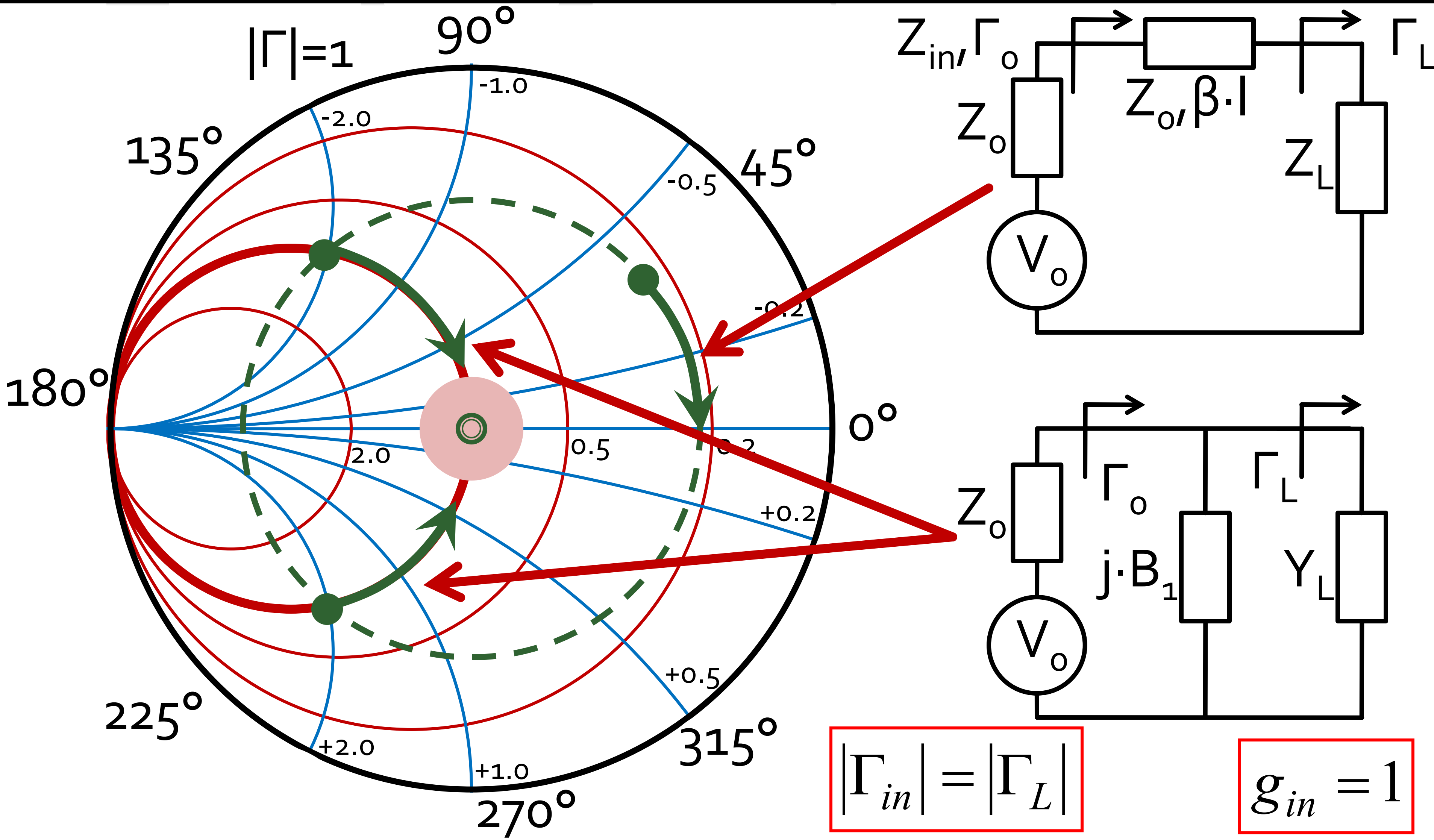
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

$$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$

  - **"-" solution**

$$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

$$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+126.35^\circ\\-126.35^\circ\end{cases}\quad \theta=\begin{cases}39.7^\circ\\93.4^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.472\\+1.472\end{cases}\quad \theta_{sp}=\begin{cases}-55.8^\circ+180^\circ=124.2^\circ\\+55.8^\circ\end{cases}$$

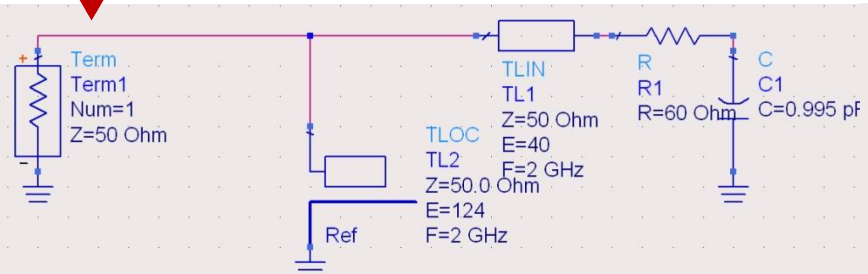
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

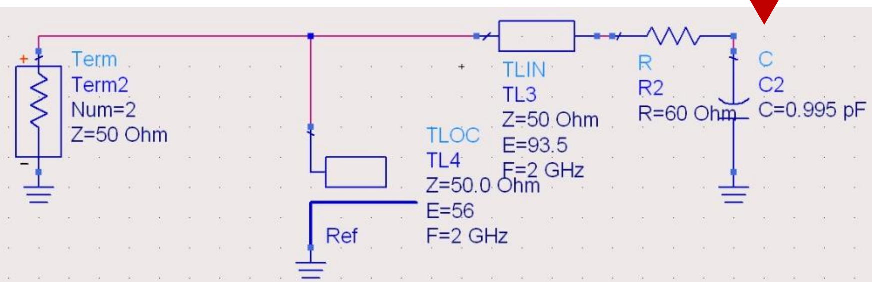
$$l_1=\frac{39.7^\circ}{360^\circ}\cdot\lambda=0.110\cdot\lambda$$

$$l_2=\frac{124.2^\circ}{360^\circ}\cdot\lambda=0.345\cdot\lambda$$

$$l_1=\frac{93.4^\circ}{360^\circ}\cdot\lambda=0.259\cdot\lambda$$

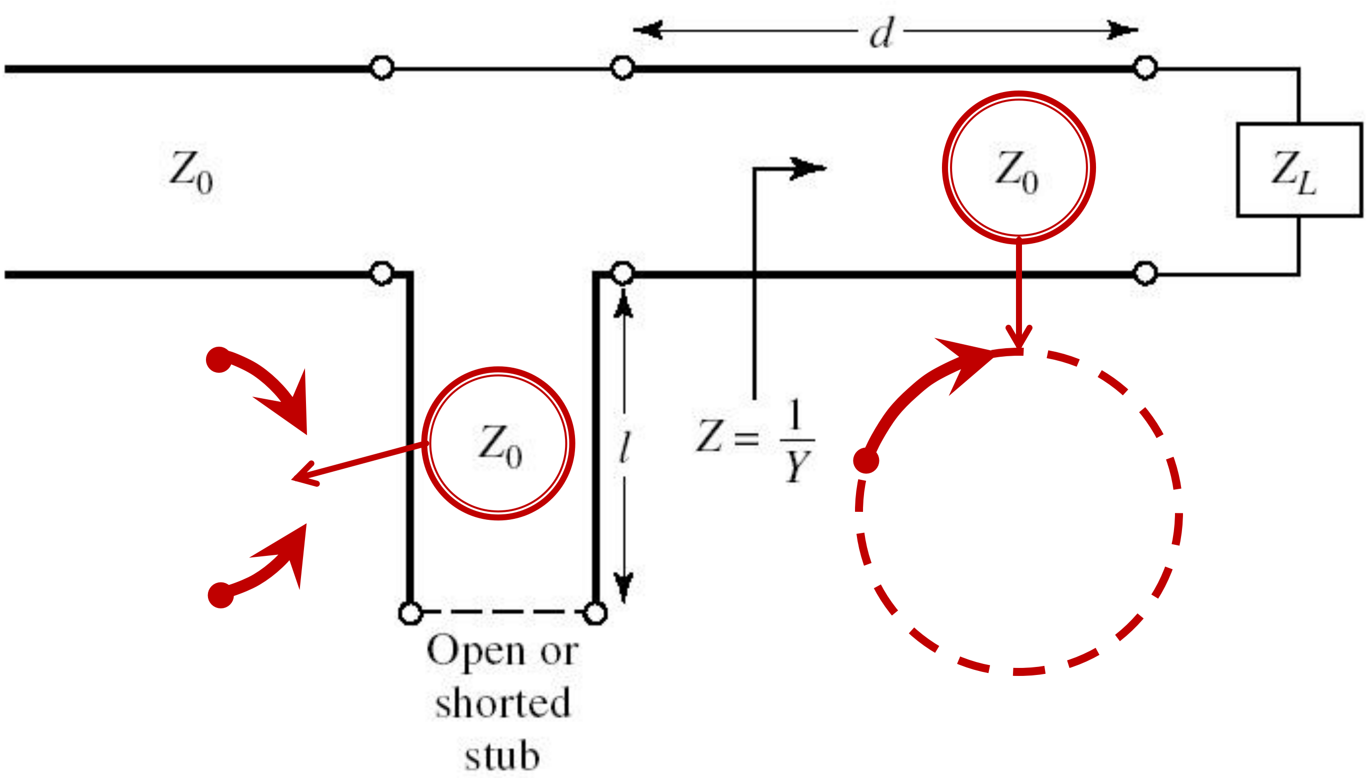
$$l_2=\frac{55.8^\circ}{360^\circ}\cdot\lambda=0.155\cdot\lambda$$



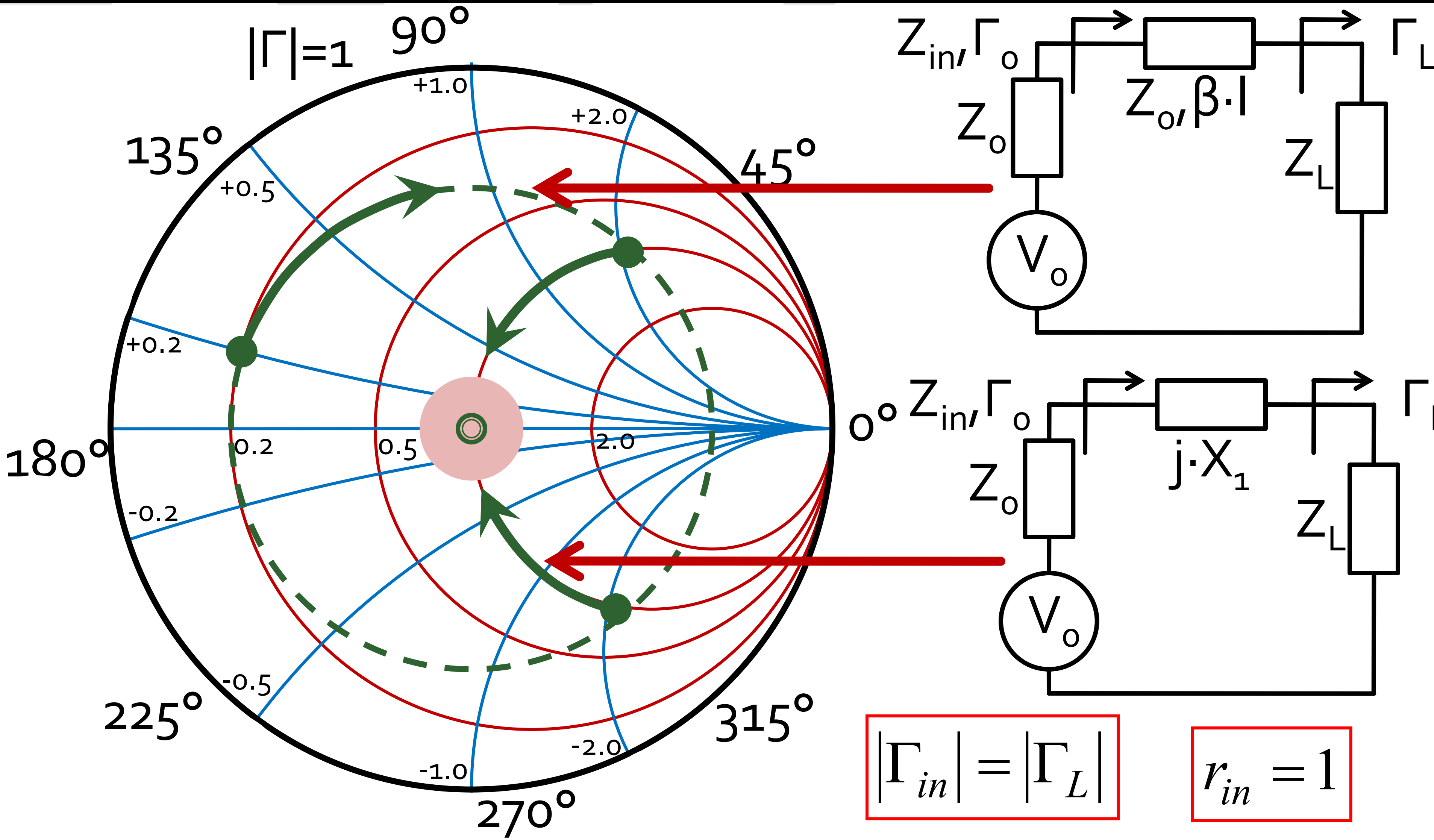


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8° (+180°) \rightarrow \theta_{ss} = 143.2°$$

  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2° (+180°) \rightarrow \theta = 166.8°$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \mathrm{Im}[z_S(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

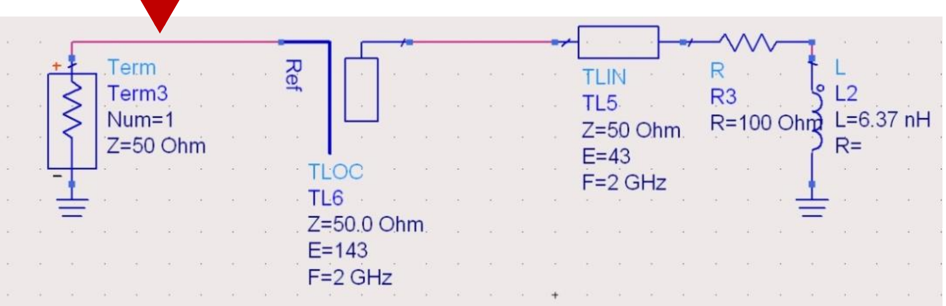
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

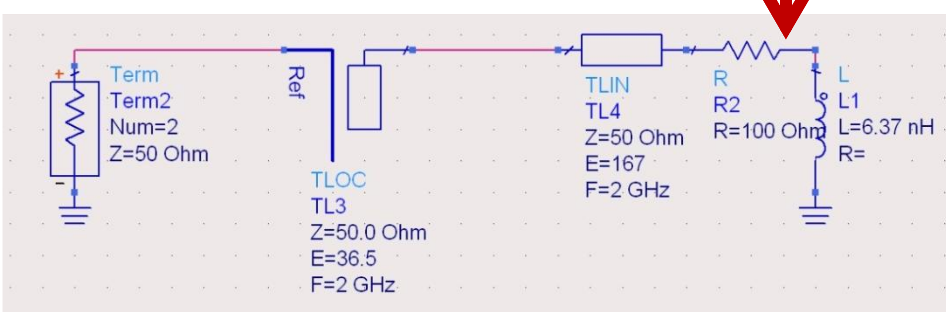
$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

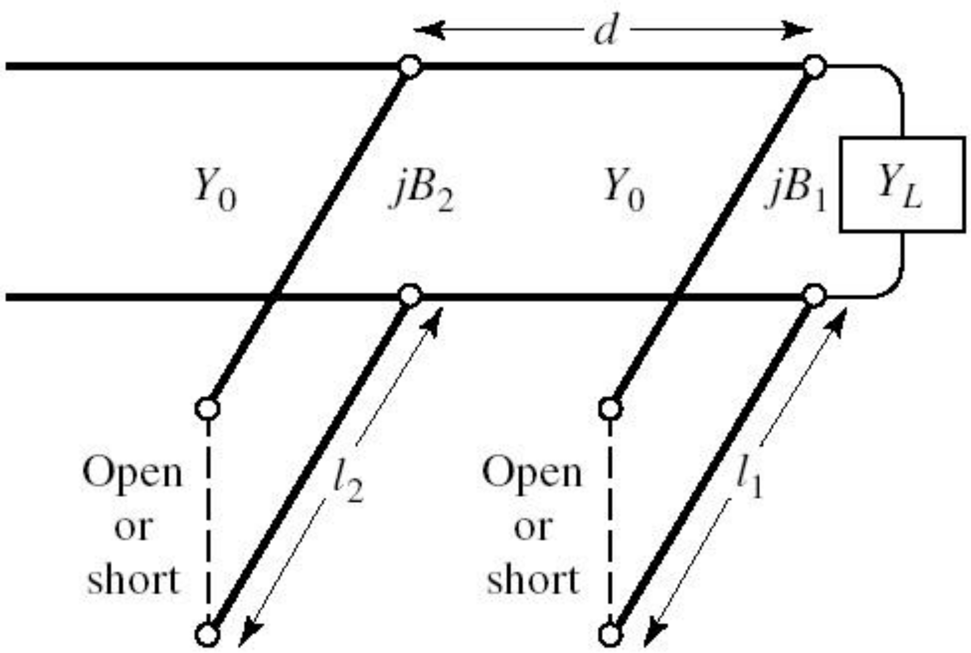
  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Double stub tuning

Adaptarea cu doua sectiuni de linie

Double stub tuning

- Double stub tuning
- uses two tuning stubs in fixed positions (a fixed length of line between the stubs)



Double stub tuning

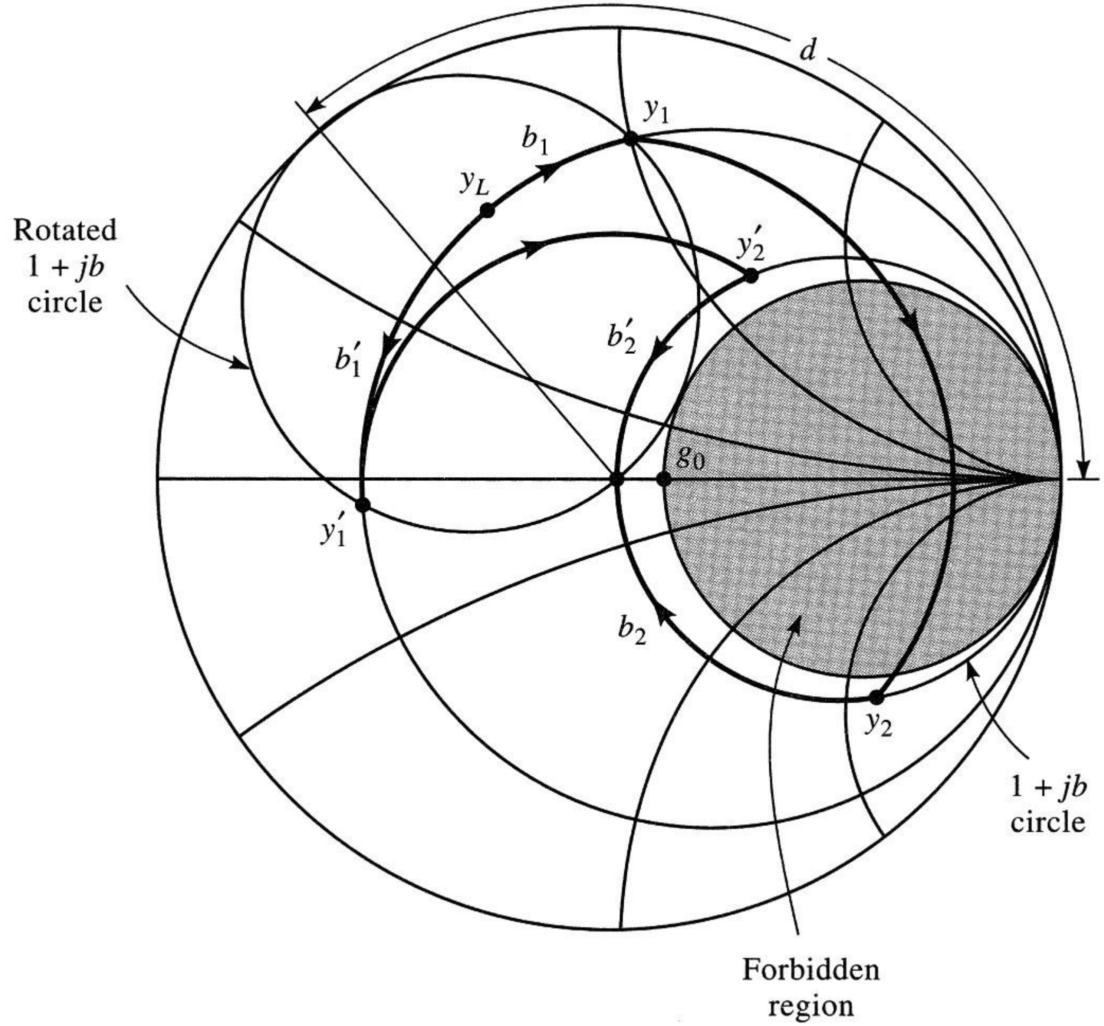
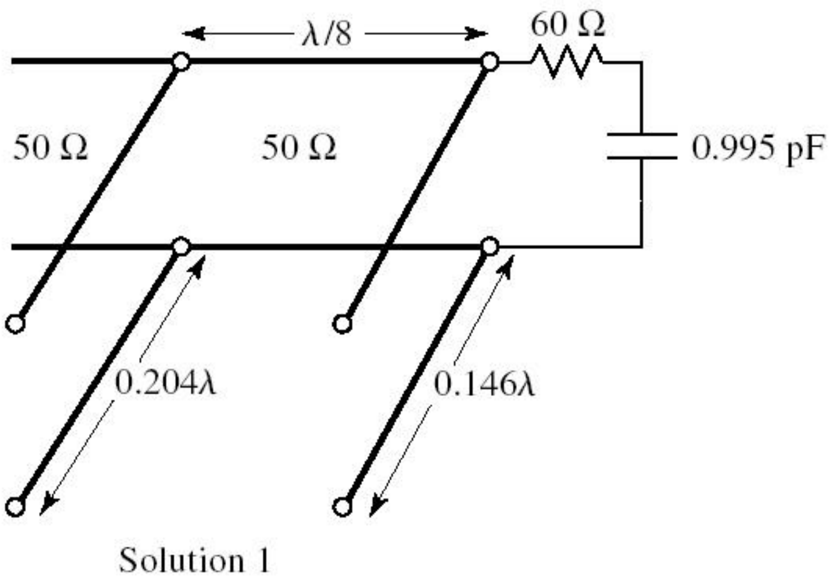
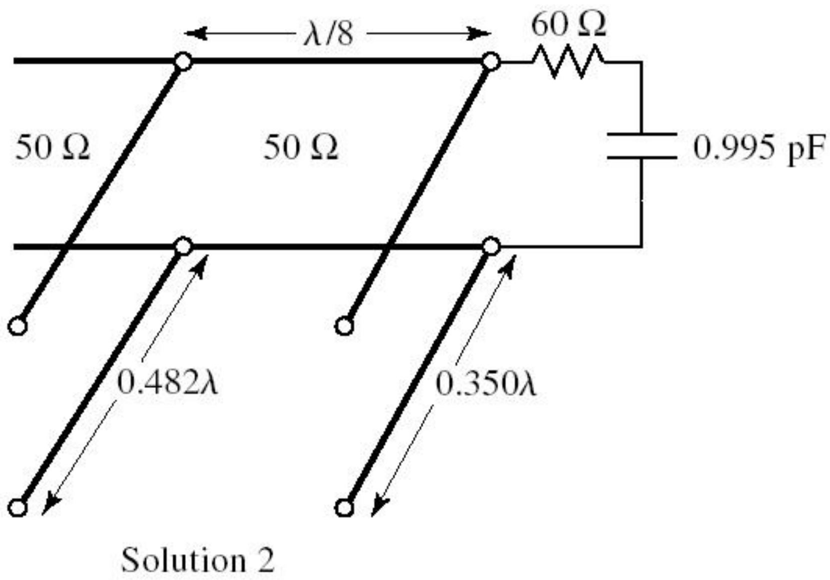


Figure 5.8

Double stub tuning

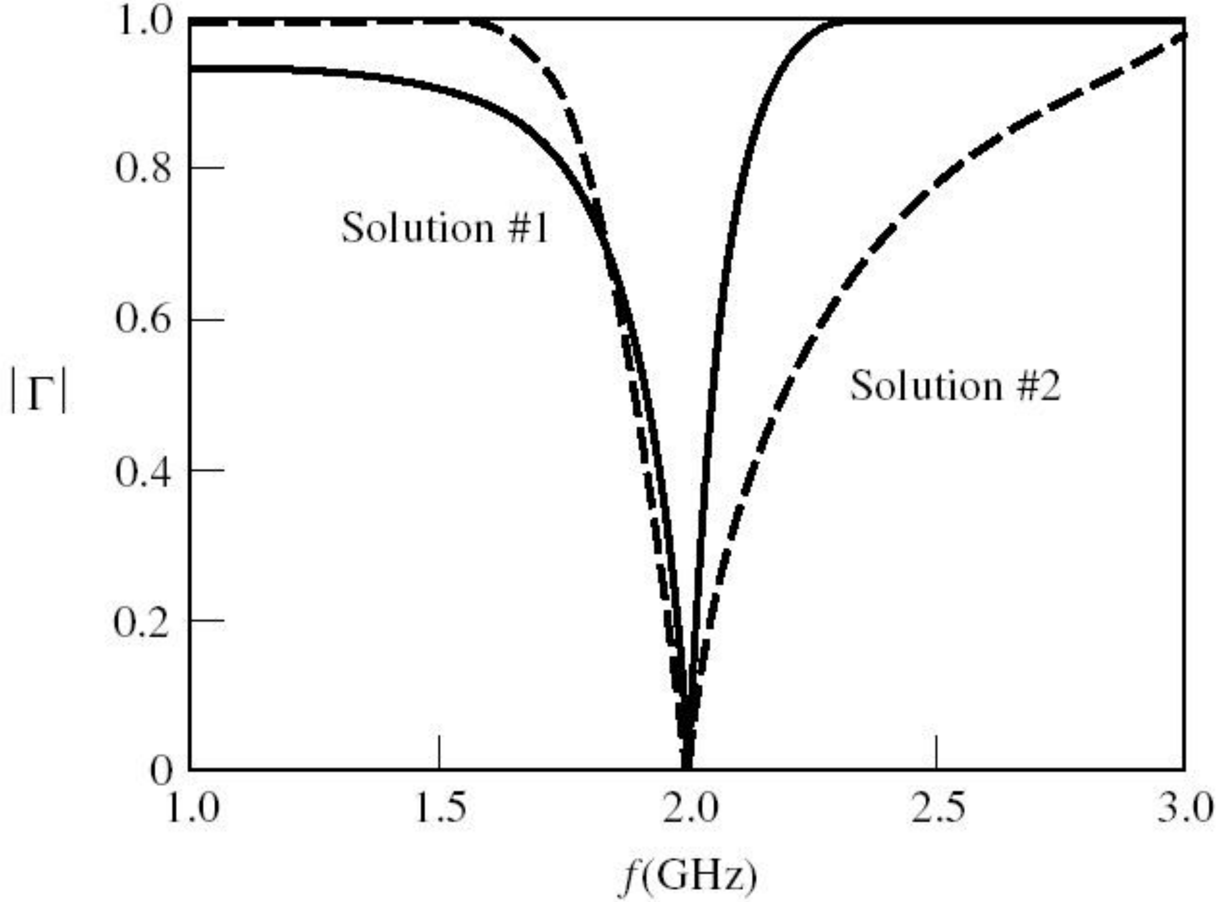
- same load: 60 Ω series with 0.995 pF at 2GHz
- two possible solutions





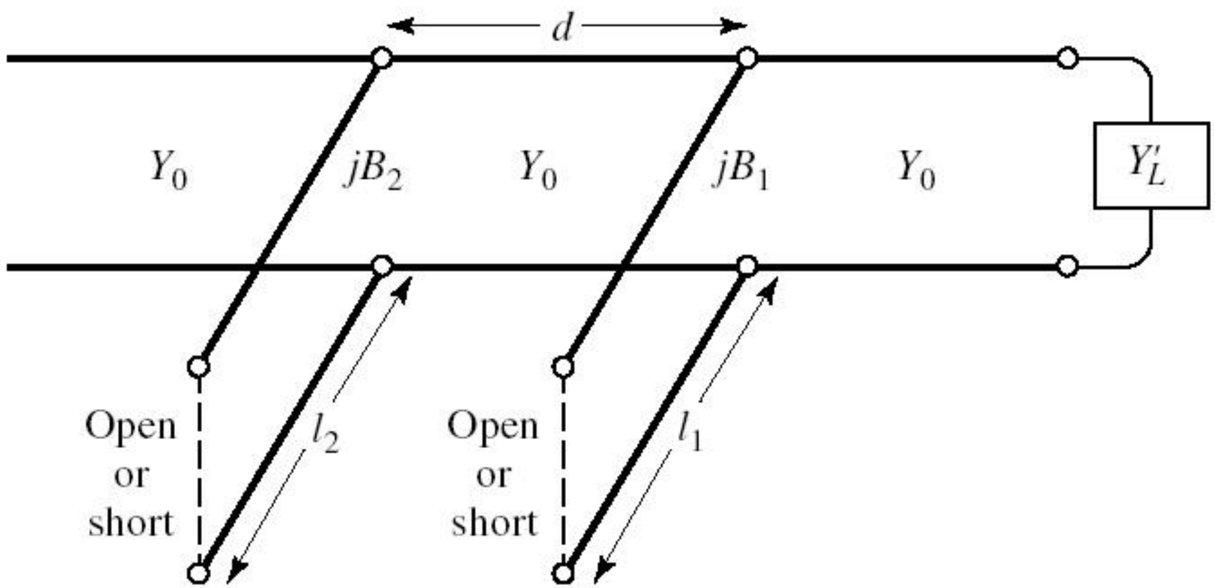
Double stub tuning

- two possible solutions

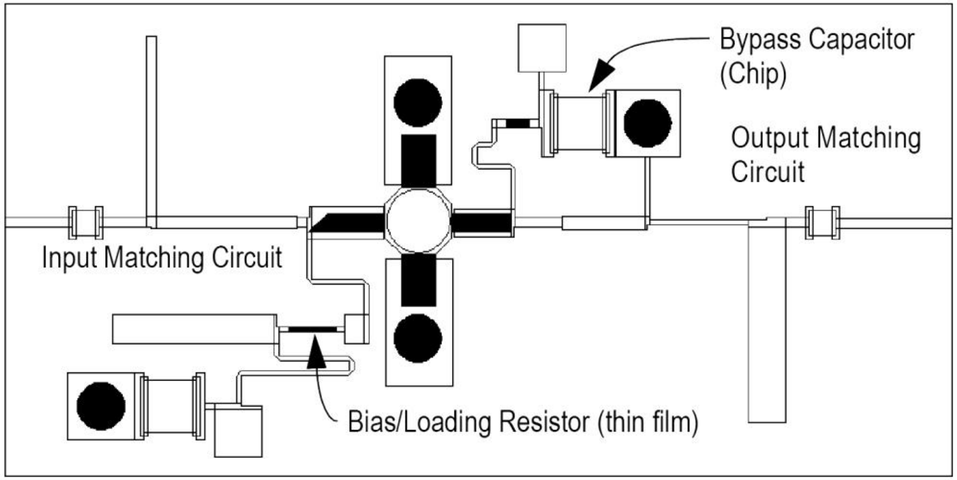


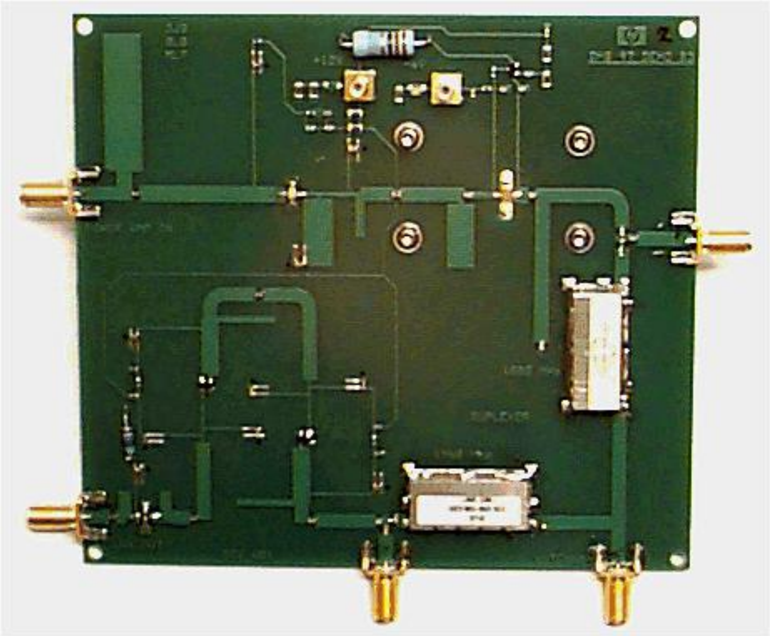
Double stub tuning

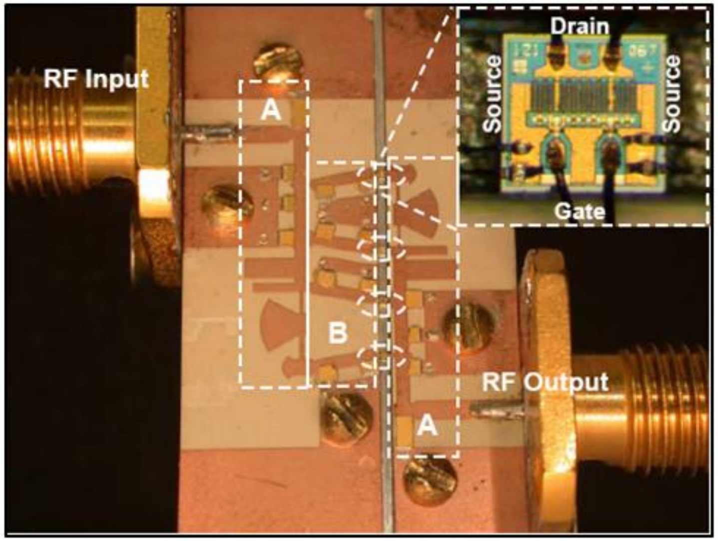
- Typically d=λ/8 or d=3λ/8
  - d -> 0 and d -> λ/2 offer frequency sensitive solutions
- **Not possible** for every load
  - unless we can add a specific length of line between the load and the first stub

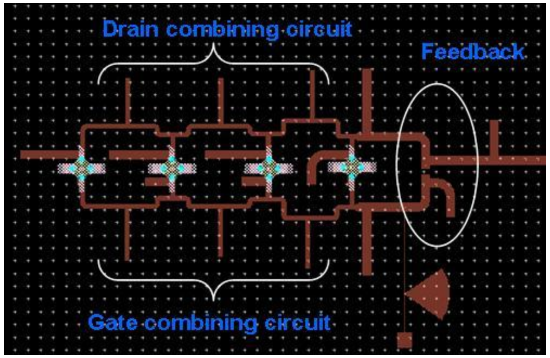


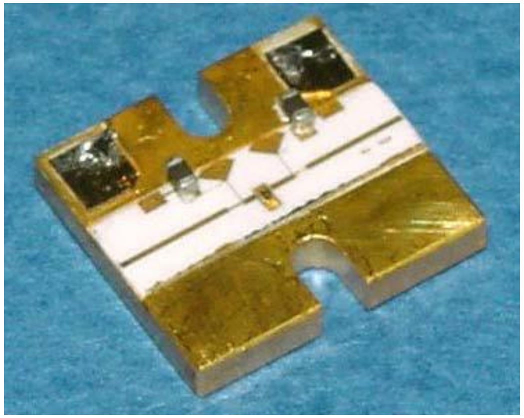
Impedance Matching with Stubs









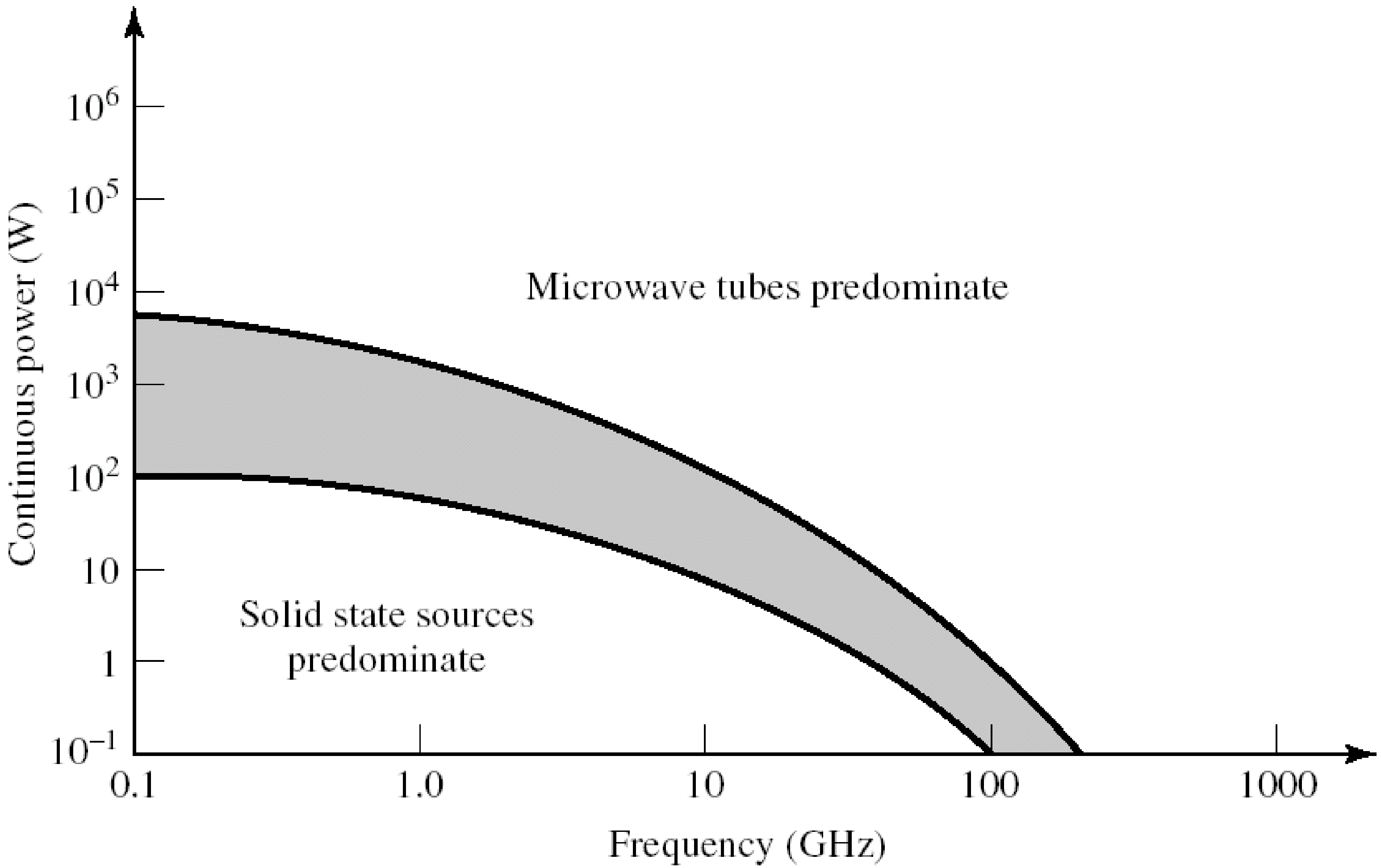


Microwave Amplifiers

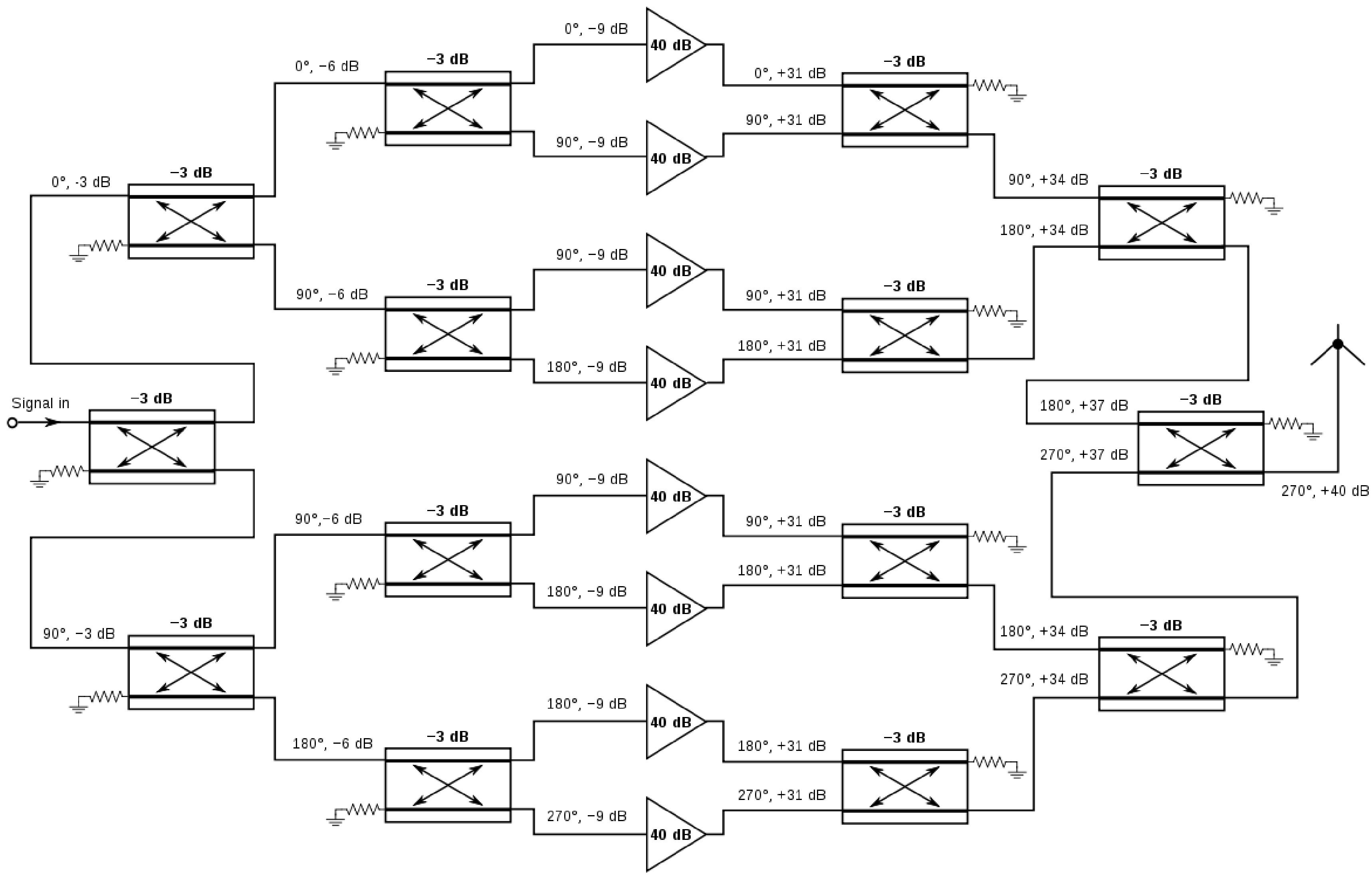
Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

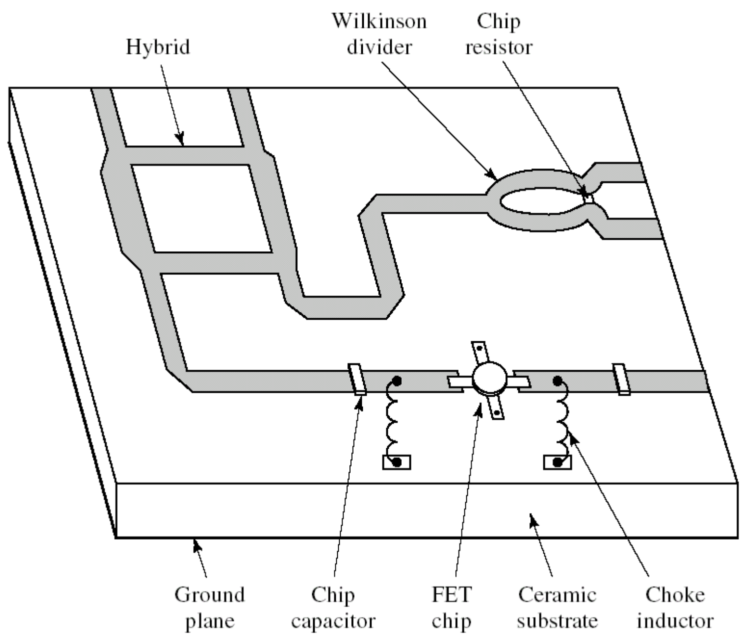
Microwave Amplifiers

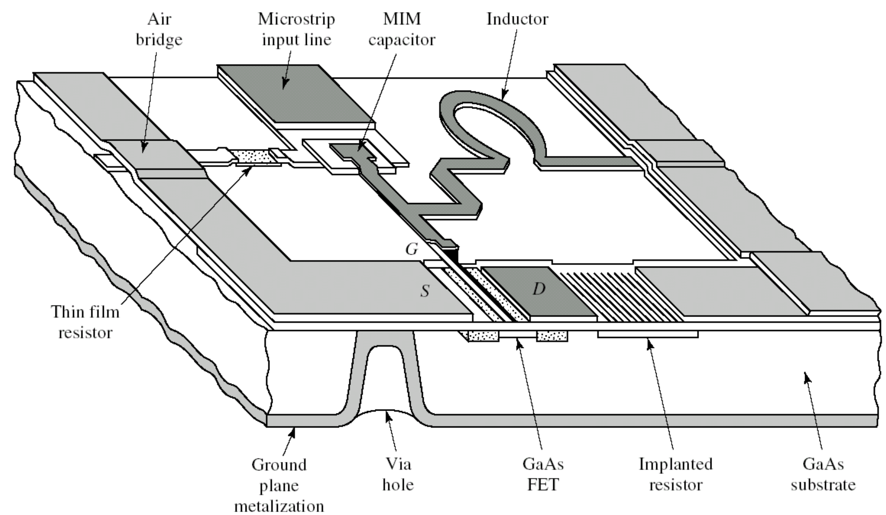


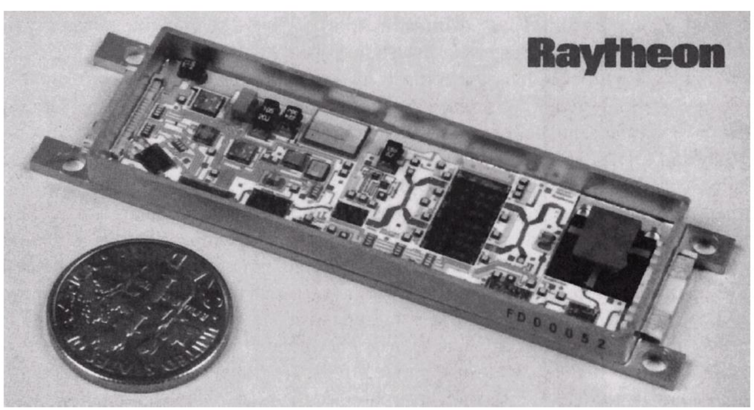
Balanced amplifiers

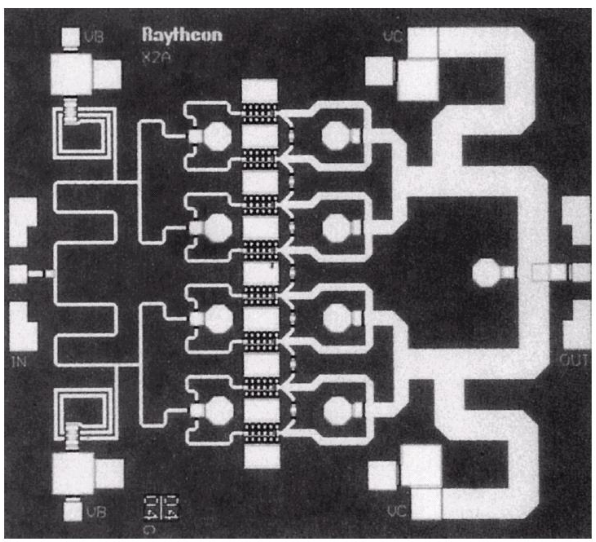


Microwave Integrated Circuits









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